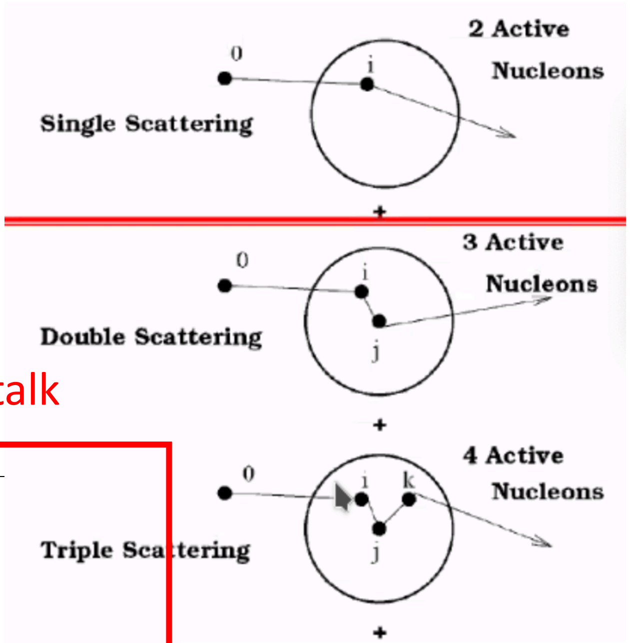
Construction of microscopic optical potentials

Andrea Idini

LUND UNIVERSITY

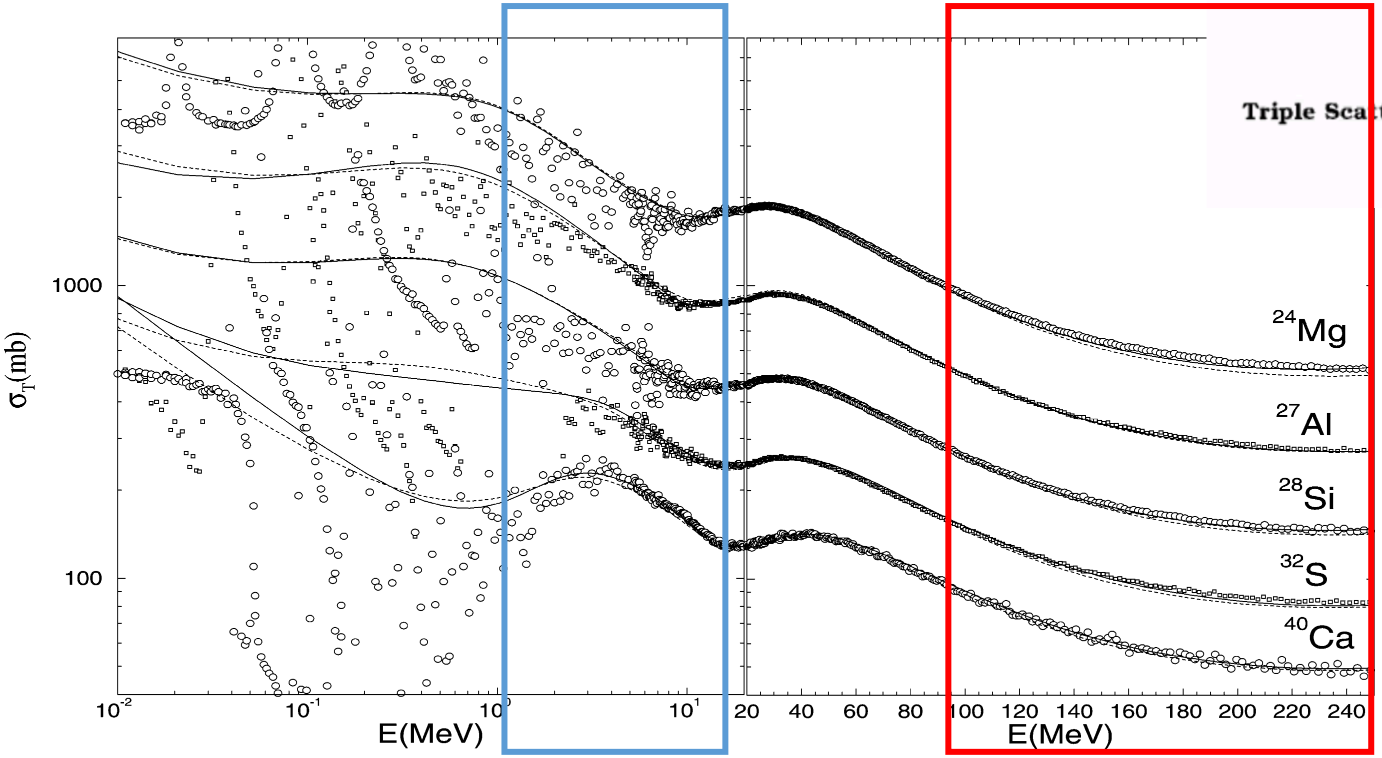
Reaction seminars series
Worldwide, 18 June 2020

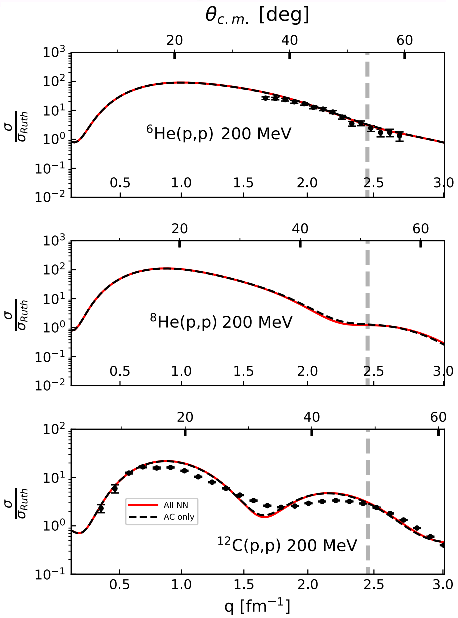
Optical potentials



This talk

Ch. Elster's talk





Koning, Delaroche, NPA713, 231 (2002)

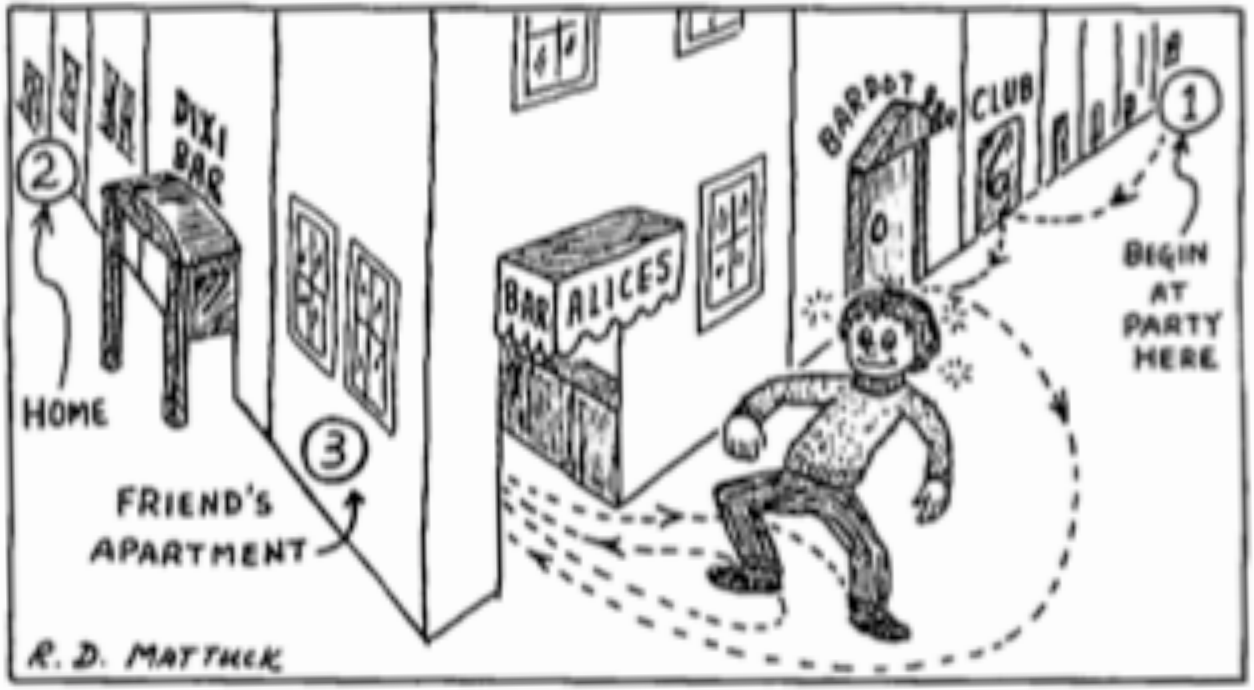


Fig. 1.1 *Propagation of Drunken Man*

(Reproduced with the kind permission of *The Encyclopedia of Physics*)

$$P(2,1) = P_0(2,1) + P_0(A,1)P(A)P_0(2,A) + P_0(B,1)P(B)P_0(2,B) + \cdots$$
$$+ P_0(A,1)P(A)P_0(B,A)P(B)P_0(2,B) + \cdots. \qquad (1.1)$$

A Guide to Feynman Diagrams in the Many-Body Problem – R.D. Mattuck

Green's functions

$$g_{\alpha\beta}(\omega+i\eta)=\sum_n \frac{\langle\psi_0^A|c_\alpha|\psi_n^{A+1}\rangle\langle\psi_n^{A+1}|c_\beta^+|\psi_0^A\rangle}{\omega-E_n^{A+1}+E_0^A+i\eta}+\sum_i \frac{\langle\psi_0^A|c_\alpha^+|\psi_i^{A-1}\rangle\langle\psi_i^{A-1}|c_\beta\,|\psi_0^A\rangle}{\omega-E_0^A+E_i^{A-1}-i\eta}$$

Källén–Lehmann spectral representation

Unperturbed case

$$g^0\,(\omega+i\eta)=\sum_i \frac{1}{E-\epsilon_i^{base}\pm i\eta}$$

Green's functions

$$g_{\alpha\beta}(\omega + i\eta) = \sum_n \frac{\langle \psi_0^A | c_\alpha | \psi_n^{A+1} \rangle \langle \psi_n^{A+1} | c_\beta^+ | \psi_0^A \rangle}{\omega - E_n^{A+1} + E_0^A + i\eta} + \sum_i \frac{\langle \psi_0^A | c_\alpha^+ | \psi_i^{A-1} \rangle \langle \psi_i^{A-1} | c_\beta \, | \psi_0^A \rangle}{\omega - E_0^A + E_i^{A-1} - i\eta}$$

Källén–Lehmann spectral representation

Unperturbed case

$$g^0\,(\omega + i\eta) = \sum_i \frac{1}{E - \epsilon_i^{base} \pm i\eta}$$

self-consistent Green's functions method finds spectra of the Hamiltonian operator

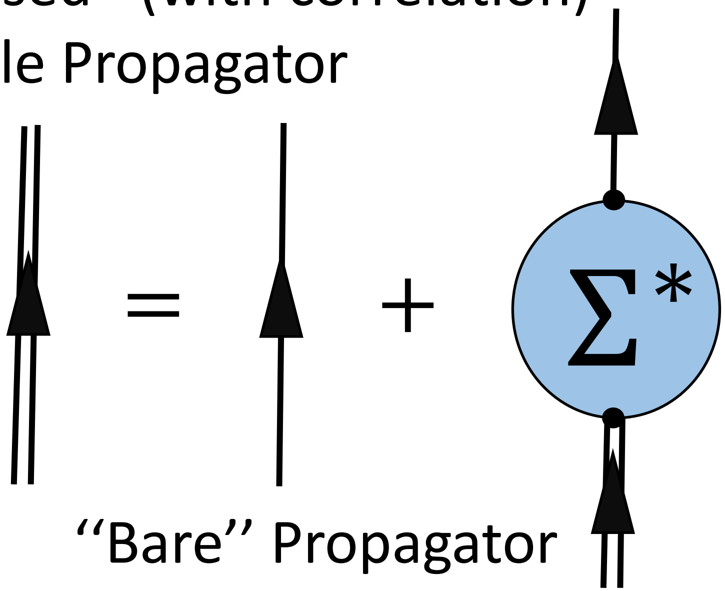
$$H(A) = T - T_{c.m.}(A+1) + V + W$$

Green's functions as many-body method

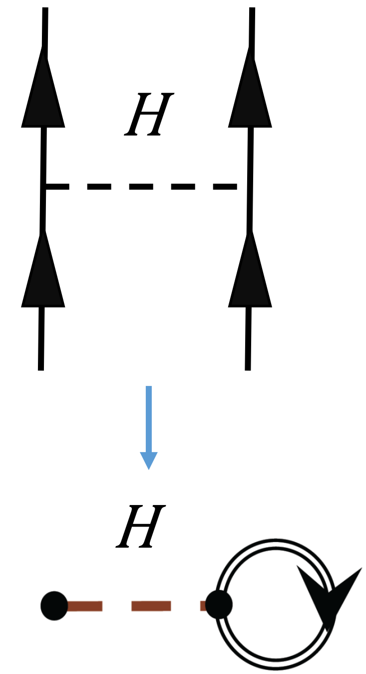
Dyson Equation

$$g(\omega + i\eta) = \; g^{0}\,(\omega + i\eta) + g^{0}(\omega + i\eta)\Sigma^{*}(\omega + i\eta)g(\omega + i\eta)$$

"Dressed" (with correlation)
Particle Propagator



$$H(A) = T - T_{c.m.}(A+1) + V + W$$

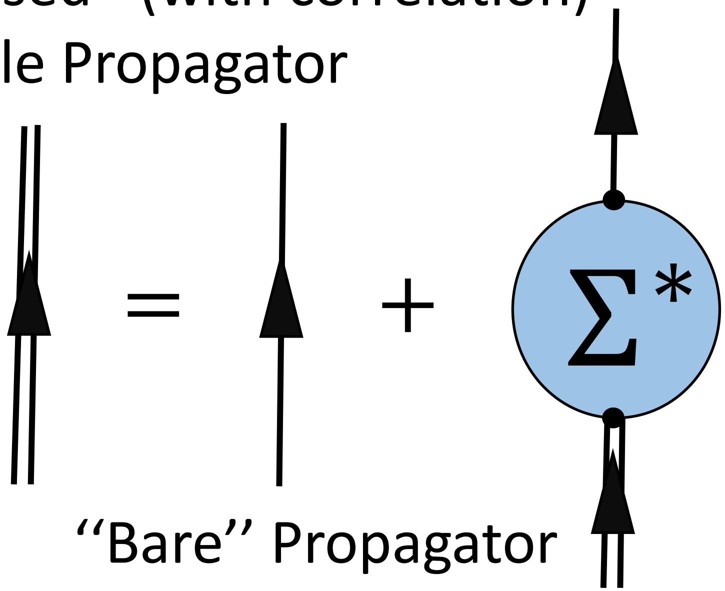


Green’s functions as many-body method

Dyson Equation

$$g(\omega + i\eta) = \ g^0\,(\omega + i\eta) + g^0(\omega + i\eta)\Sigma^*(\omega + i\eta)g(\omega + i\eta)$$

“Dressed” (with correlation)
Particle Propagator

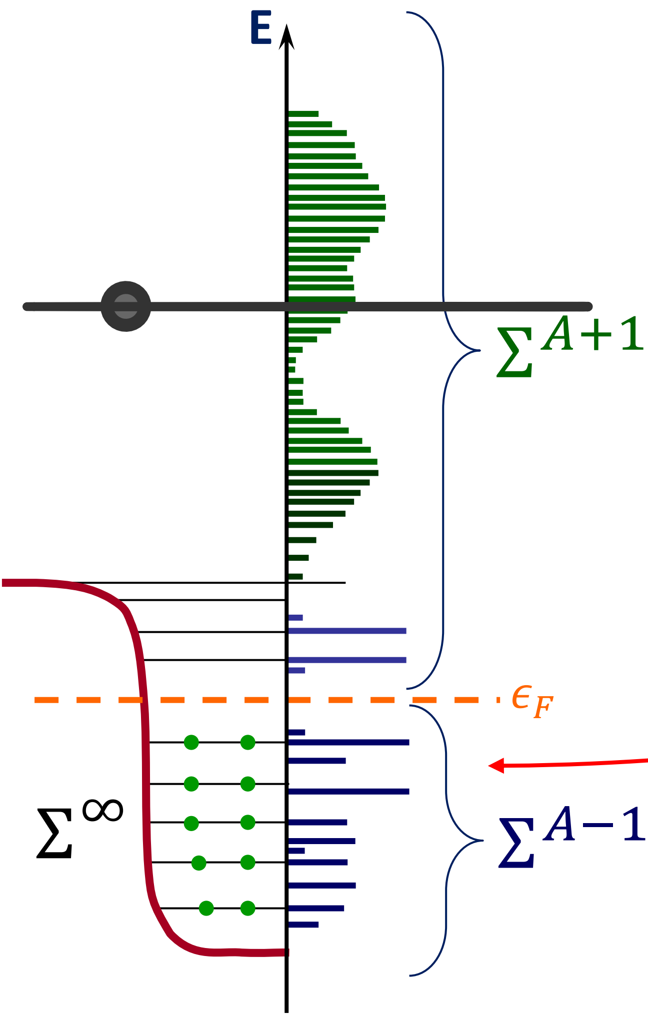


Interaction between the particle and the system (physical choice)

Fragments and **changes energy** of the “bare” state

$$\Sigma_{\alpha\beta}(\omega + i\eta) = \sum_r \frac{m_\alpha^r m_\beta^r}{\omega - E_r + i\eta}$$

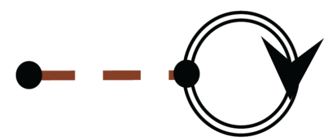
Nucleon elastic scattering



The irreducible self-energy is a nucleon-nucleus optical potential*

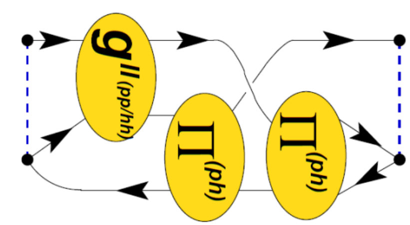
$$\Sigma^{*}_{\alpha\beta}(r,r';\omega) = \Sigma^{\infty}_{\alpha\beta} + \sum_i \frac{m^i_\alpha \, m^{i*}_\beta}{\omega - \epsilon_i \pm i\eta}$$

correlated mean-field



resonances beyond mean-field

Correlations



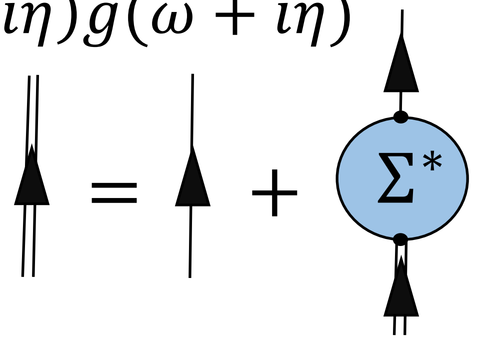
➔ This provides *consistent* many-body and scattering wave functions

*Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)

Green's functions as optical potentials

Dyson Equation

$$g(\omega + i\eta) = \; g^0\,(\omega + i\eta) + g^0(\omega + i\eta)\Sigma^*(\omega + i\eta)g(\omega + i\eta)$$



Equation of motion

$$\left(E + \frac{\hbar^2}{2m}\nabla_r^2\right) g(r,r';\, E,\Gamma) = \delta(r - r') + \int dr''\Sigma^*(r,r'';E,\Gamma)g(r'',r;E,\Gamma)$$

Corresponding Hamiltonian

$$H(r,r') = -\frac{\hbar^2}{2m}\nabla_r^2 + \;\Sigma^*(r,r';E,\Gamma)$$

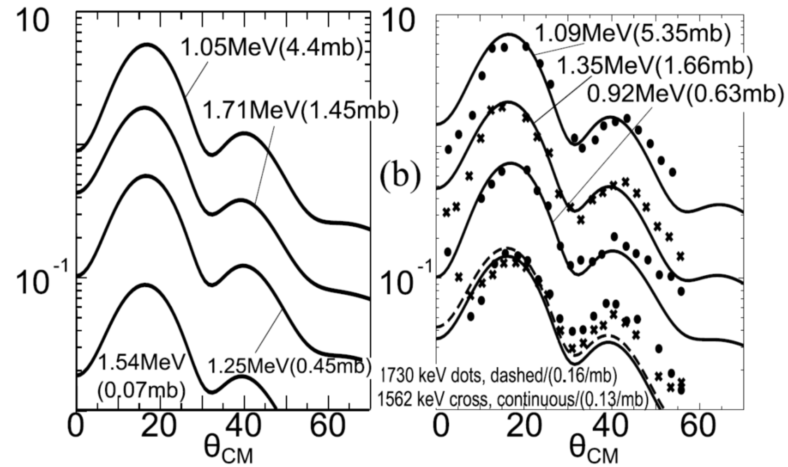
Σ corresponds to the Feshbach's generalized optical potential

Escher & Jennings PRC66 034313 (2002)

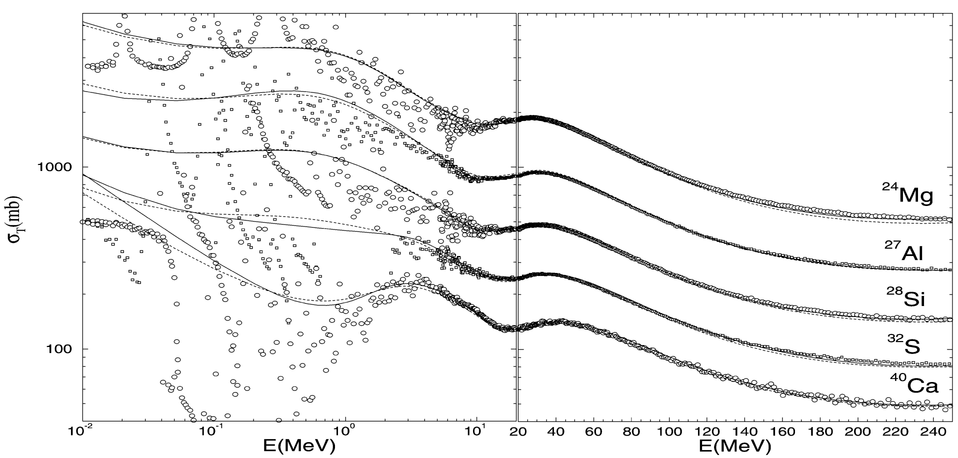
Why optical potentials?

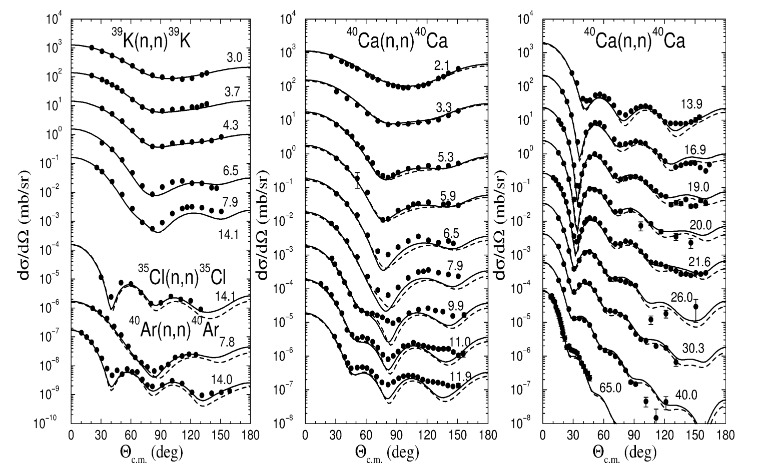
- Optical potentials **reduce many-body complexity** decoupling structure contribution and reactions dynamics.
- Often fitted on elastic scattering data (locally or globally)

1 particle transfer



A.I. et al. PRC **92**, 031304 (2015)



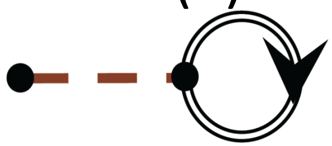


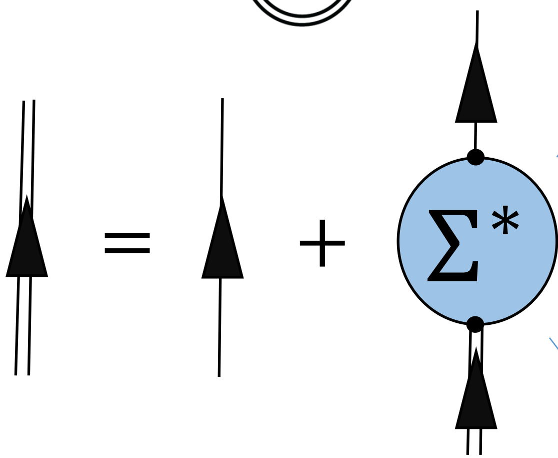
Koning, Delaroche, NPA713, 231 (2002)

Green functions and Dyson equation

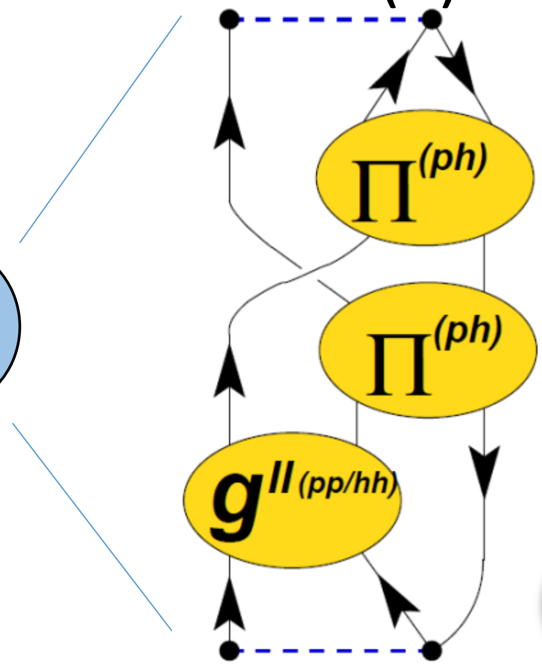
$$g_{\alpha\beta}(\omega) = g^0_{\alpha\beta}(\omega) + \sum_{\gamma\delta} g^0_{\alpha\gamma}(\omega)\Sigma^\star_{\gamma\delta}(\omega) g_{\delta\beta}(\omega)$$

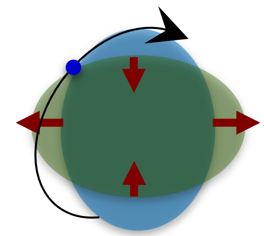
HF
ADC(1)



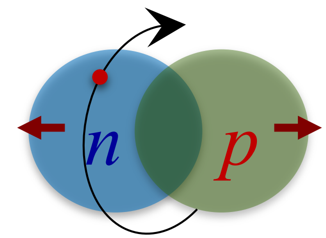


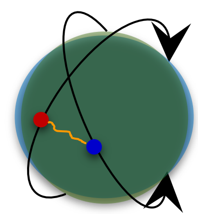
Faddeev RPA
ADC(3)





Particle hole
'polarization'
propagator
(ph-RPA)

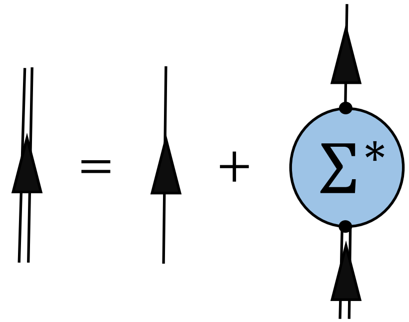


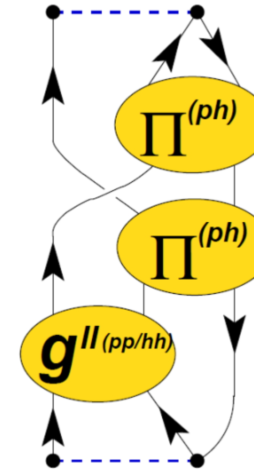


Particle-particle (pp-RPA)
two-body correlation 'ladder' propagator

Källén–Lehmann spectral representation

$$H(A) = T - T_{c.m.}(A+1) + V + W$$





$$g_{\alpha,\beta}(E,\Gamma) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - E_n^{A+1} + E_0^A + i\Gamma} + \sum_i \frac{\langle \psi_0^A | c_\alpha^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | c_\beta | \Psi_0^A \rangle}{E - E_0^A + E_i^{A-1} - i\Gamma},$$

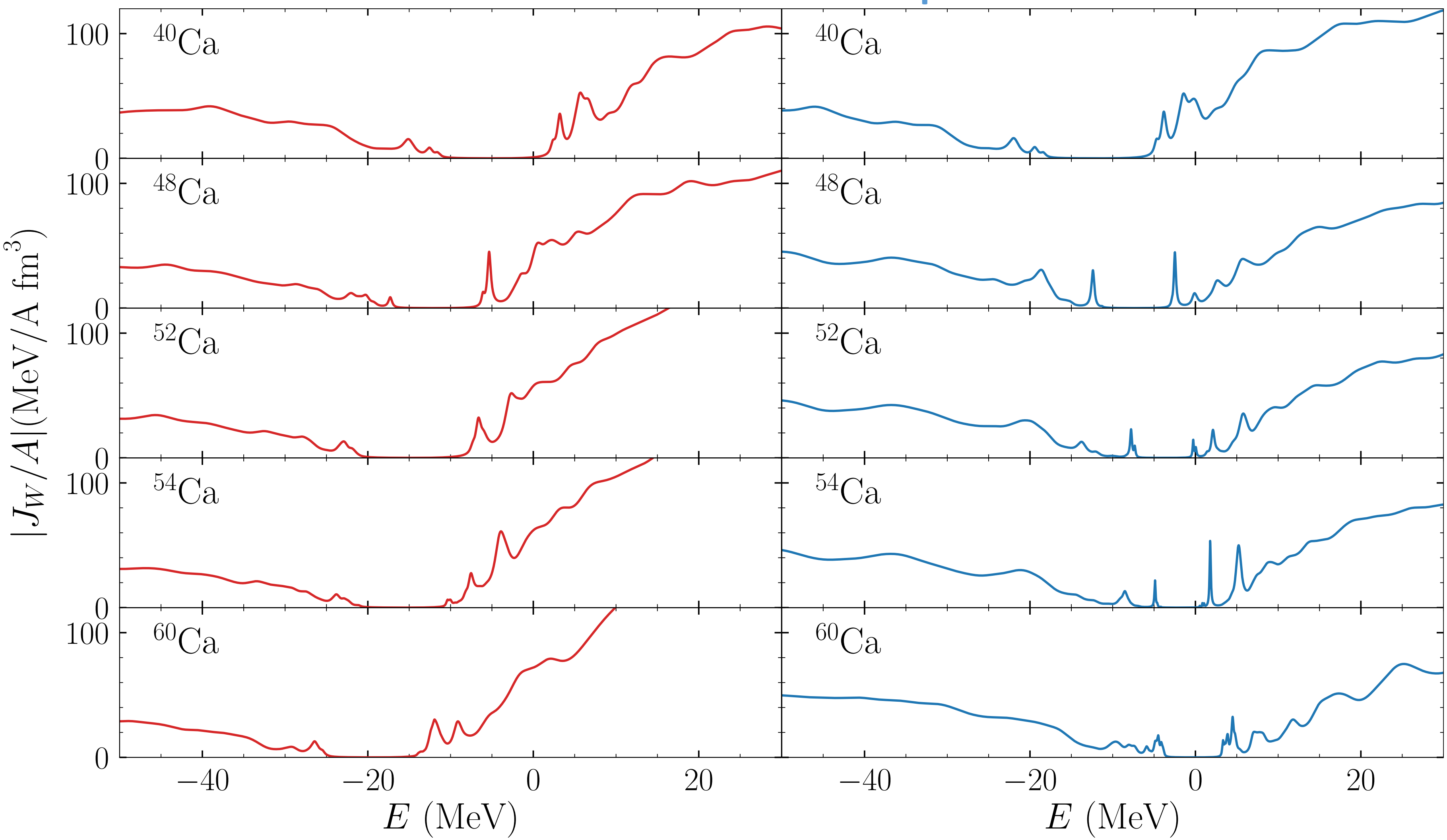
Overlaps of
A+1 and A-1 states

Excited states calculated from Dyson equation

Volume integrals

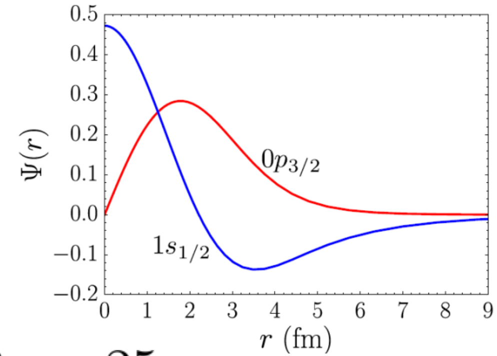
$$J_W^{\ell}(E) = 4\pi \int dr r^2 \int dr' r'^2 \mathrm{Im}\ \Sigma_0^{\ell}(r, r'; E) \qquad \tilde{\Sigma}^{\ell j}_{n_a,n_b}(E) = \sum_r \frac{m^r_{n_a} m^r_{n_b}}{E - \varepsilon_r \pm i\eta}$$

Non local potential

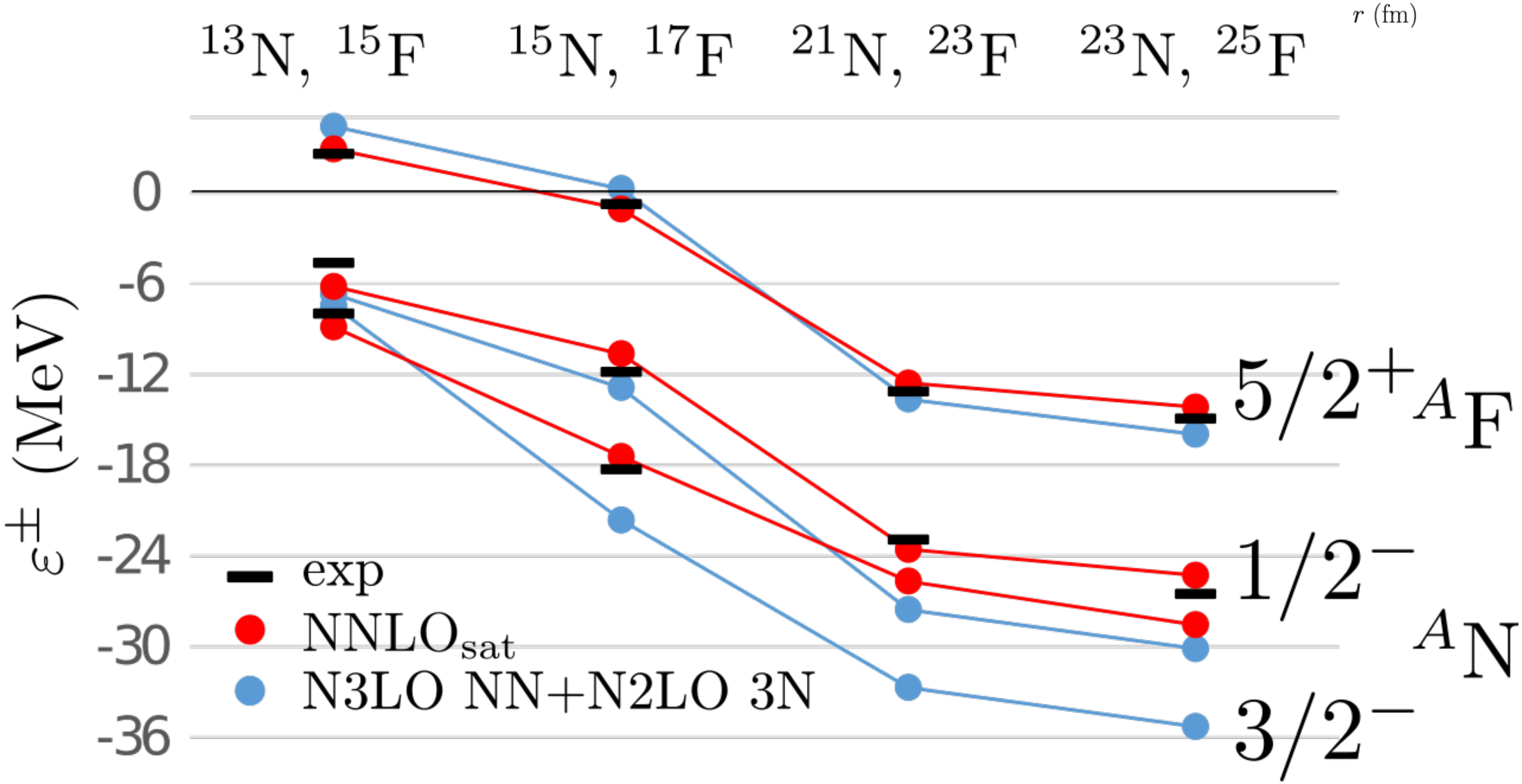


Overlap function

$$\Psi_i(r) = \sqrt{A}\int dr_1 \underset{r_i}{\cancel{\dots}} dr_A \Phi^{+}_{(A-1)}(r_1, \underset{r_i}{\cancel{\dots}}, r_{A-1})\Phi^{+}_{(A)}(r_1, \dots, r_A)$$



Proton particle-hole gap

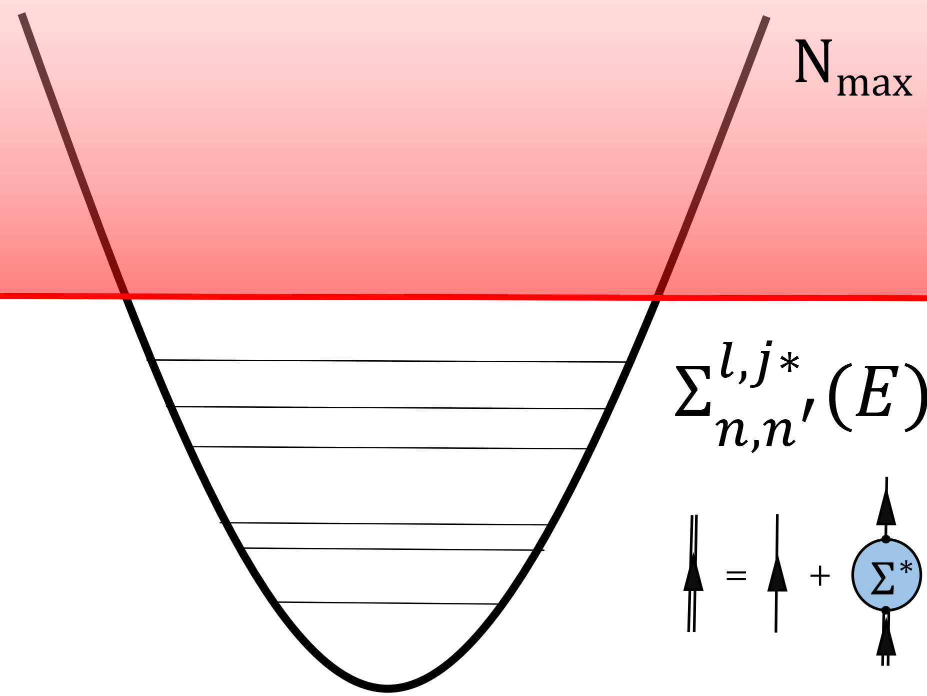


EM results from A. Cipollone PRC**92**, 014306 (2015)

- Solve Dyson equation in HO Space, find $\Sigma^{l,j*}_{n,n'}(E)$

- diagonalize in full continuum momentum space $\Sigma^{l,j*}(k,k',E)$

$$\frac{k^2}{2\gamma m}\psi_{l,j}(k) + \gamma^3 \int dk'k'^2 \left(\Sigma^{l,j*}(\gamma k, \gamma k', \gamma E)\right)\psi_{l,j}(k') = \mathrm{E}\,\psi_{l,j}(k)$$

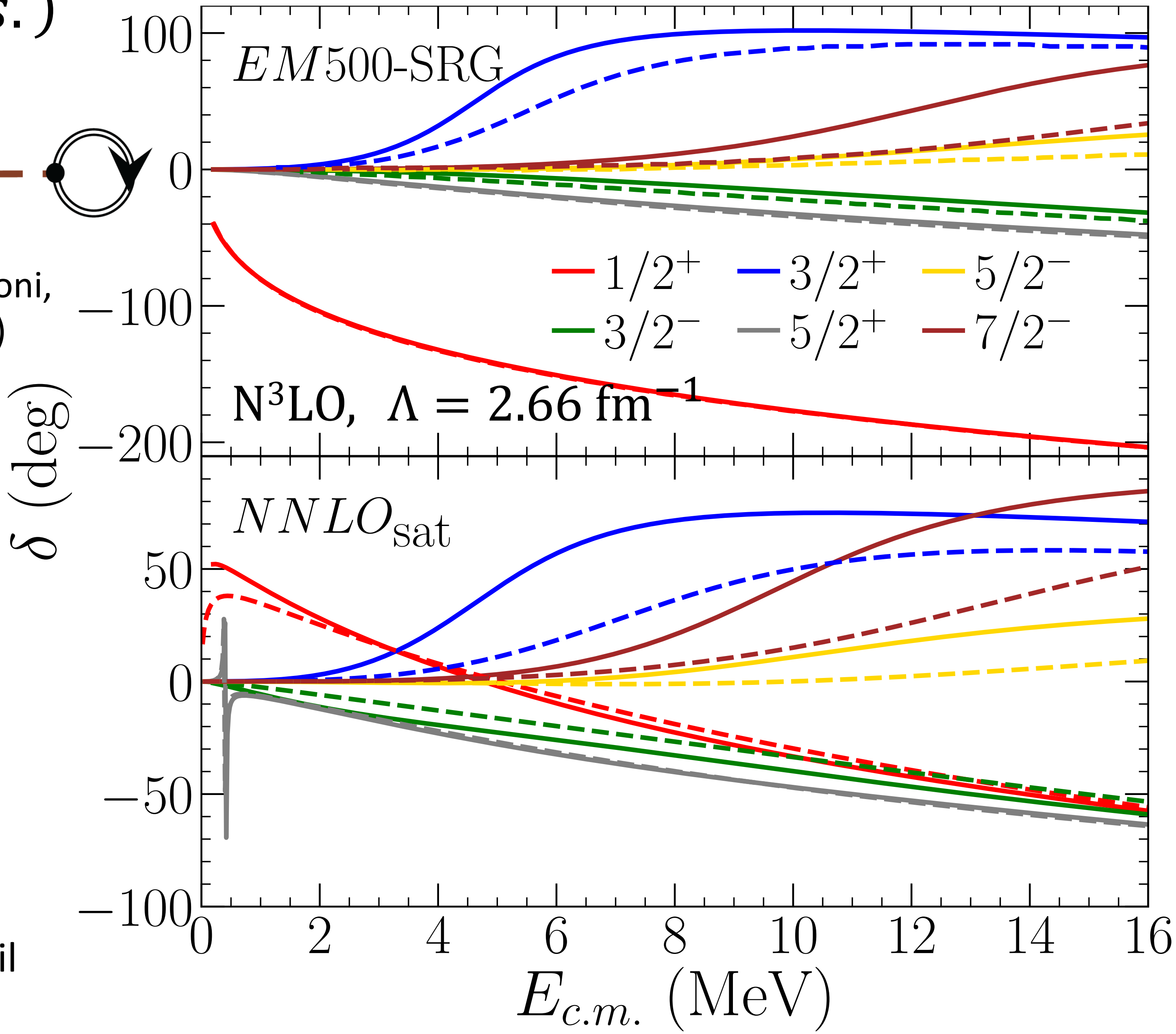


$n + {}^{16}\mathrm{O}\ (g.s.)$

NCSMC

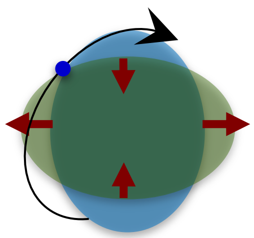
Σ^{∞}

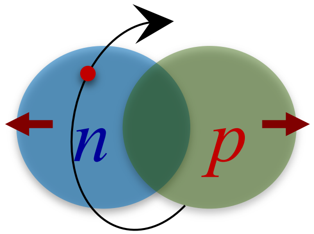
Navràtil, Roth, Quaglioni, PRC82, 034609 (2010)

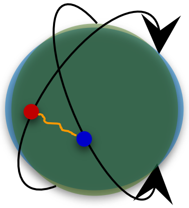


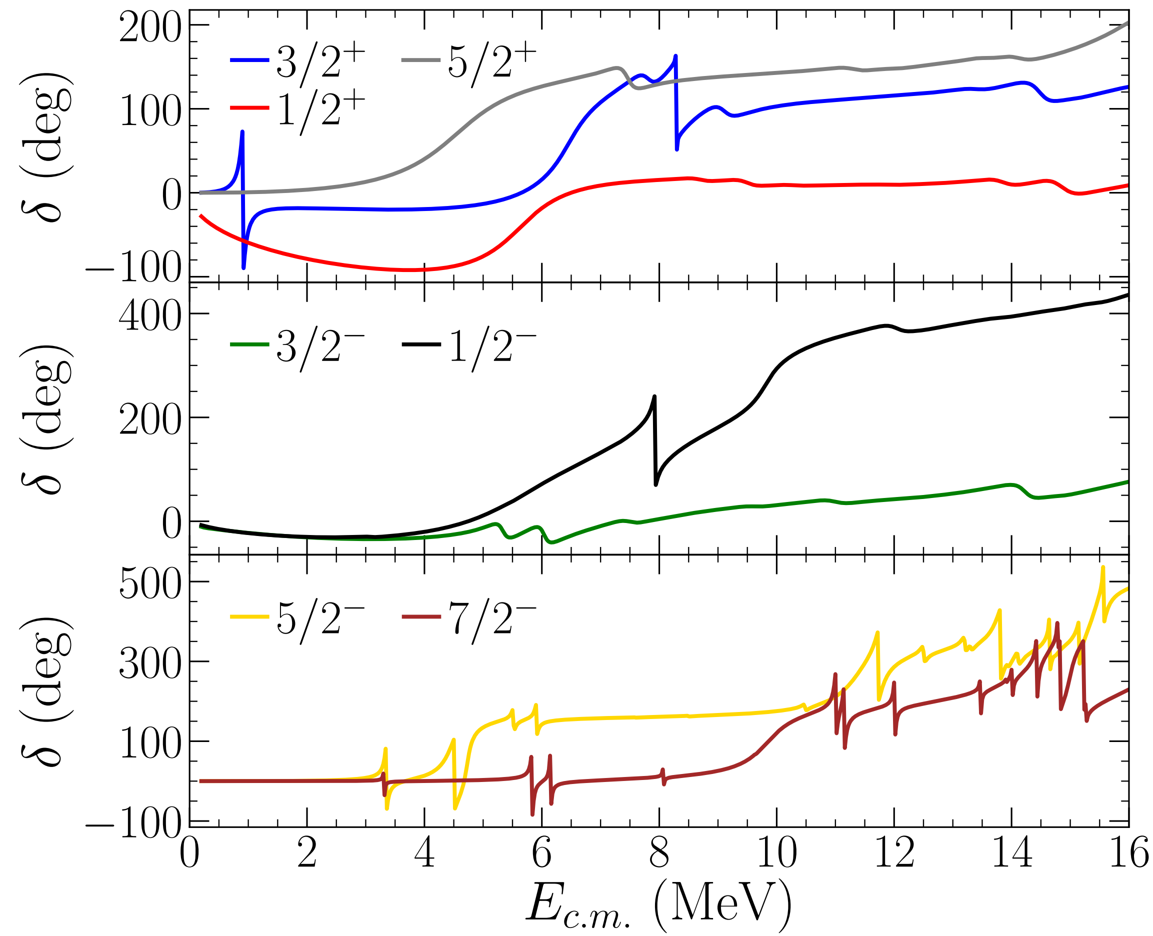
AI, Barbieri, Navràtil
PRL 123, 092501

NNLO$_{\text{sat}}$ $n + {}^{16}\text{O}\,(g.s. + exc)$



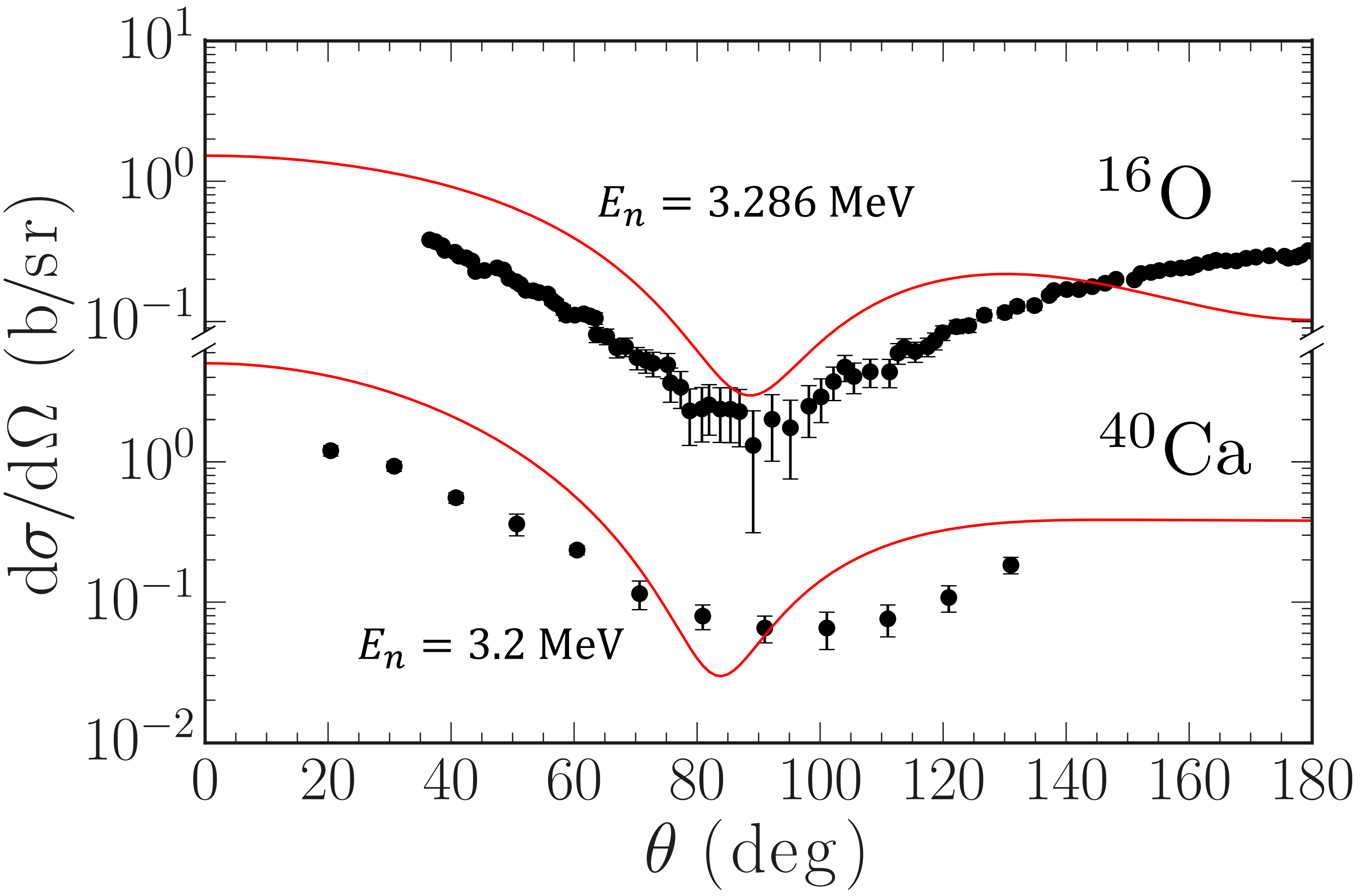




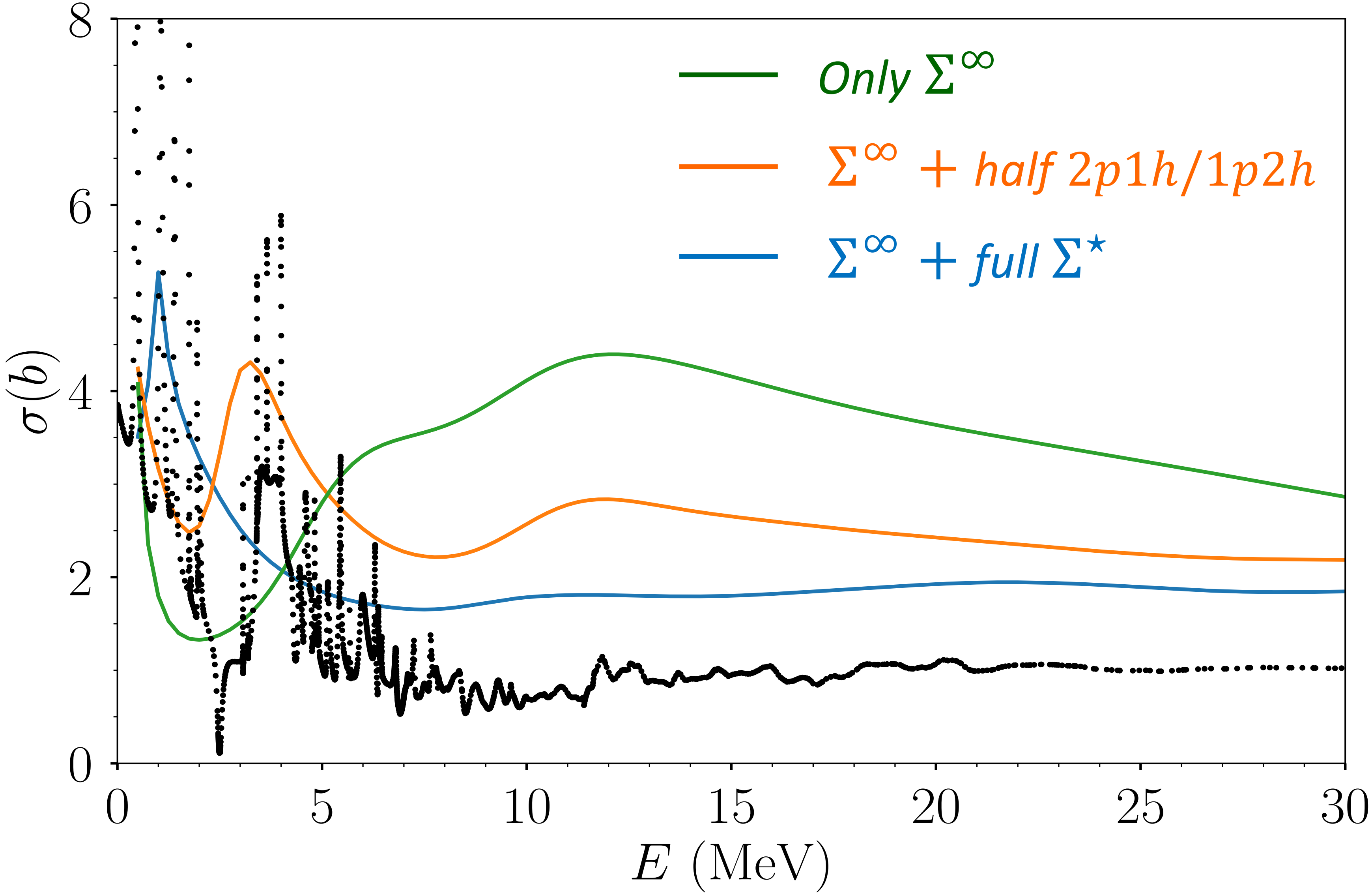


ε (MeV)	$5/2^+$	$1/2^+$	$1/2^-$	$5/2^-$	$3/2^-$	$3/2^+$	$5/2^+_*$	$5/2^-_*$	$7/2^-_*$
exp.	-4.14	-3.27	-1.09	-0.30	0.41	0.94	3.23	3.02	3.54
NNLO$_{\text{sat}}$	-5.06	-3.58	-0.15	-1.23	-2.24	0.91	4.57	3.36	3.37

neutron elastic scattering from ab initio optical potential



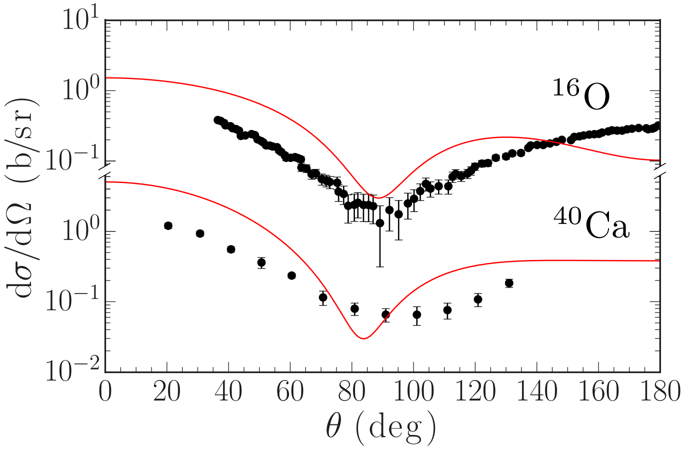
^{16}O+n

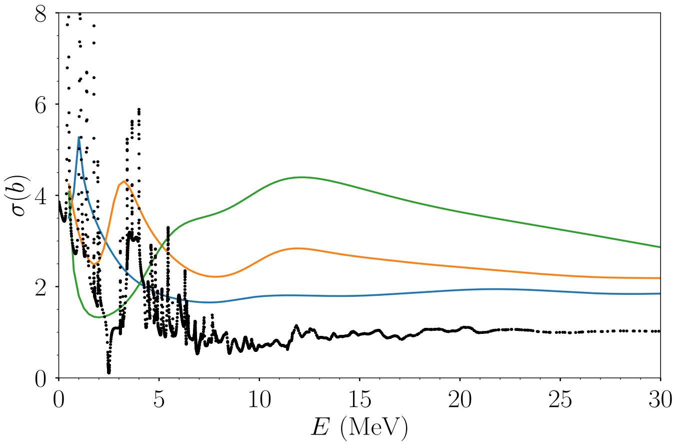


Only Σ^{∞}
Σ^{∞} + half 2p1h/1p2h
Σ^{∞} + full $\Sigma^{\star}$
$\sigma(b)$
E (MeV)
0
2
4
6
8
0
5
10
15
20
25
30

Conclusions

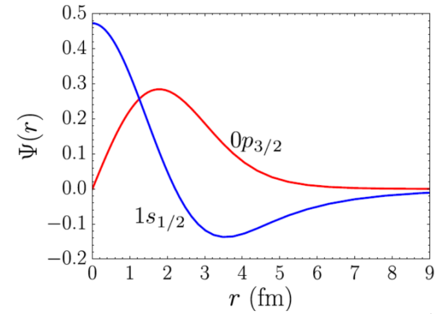
- We are developing an interesting tool to study nuclear reactions effectively: a non-local generalized optical potential corresponding to nuclear self energy.
- SCGF provide a rich description of low energy properties.
- (p-h) correlations are related to absorption, *that is missing*

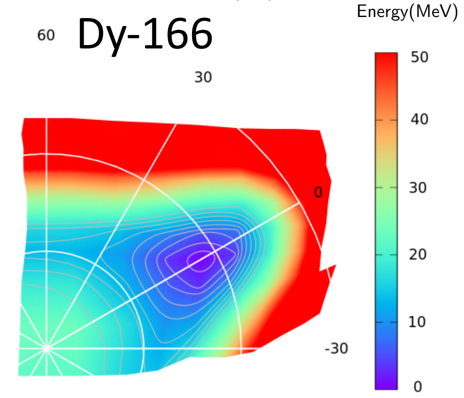




Perspectives

- Use the information of SCGF in the continuum in other contexts: e.g. overlap functions for Knockout
- Explore the effect of different bases and bridge the Energy gap between spectator and GF expansions
- Enrich the description of correlations in ground and excited states: *multiconfiguration with projection*





Thanks to

Surrey

- C. Barbieri

TRIUMF

- P. Navrátil

Lund

- J. Ljungberg
- J. Rotureau
- G. Carlsson

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The Crafoord Foundation
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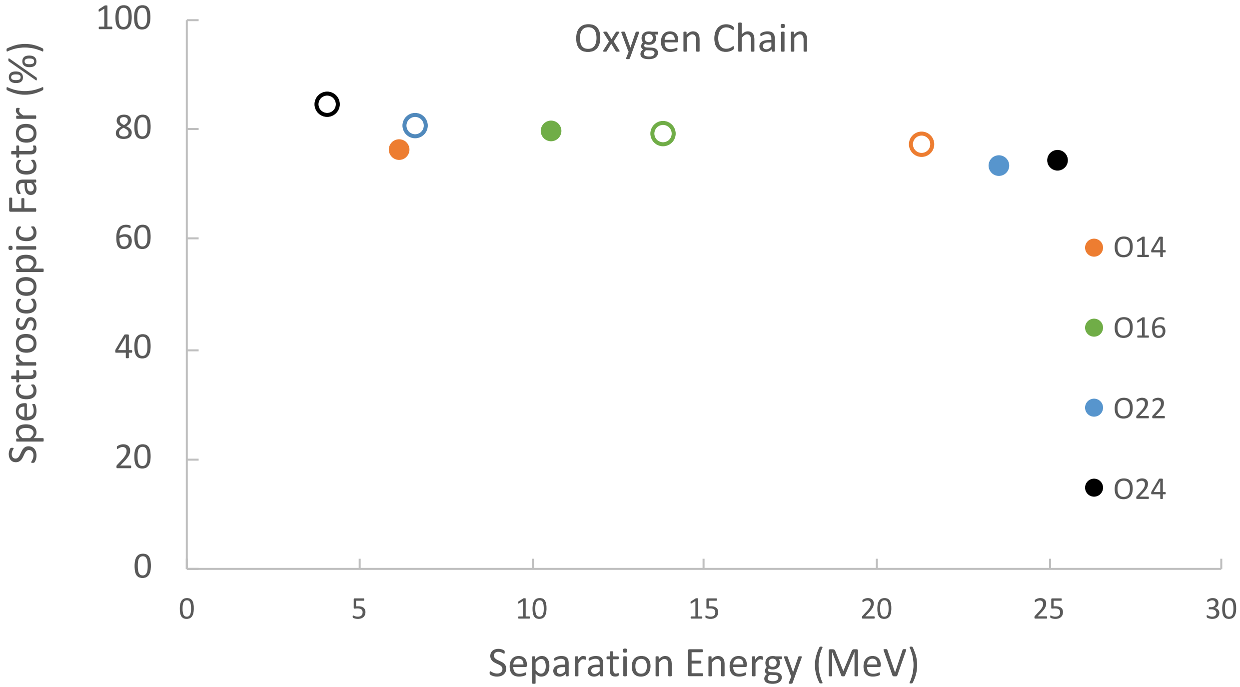
Knockout Spectroscopic Factors

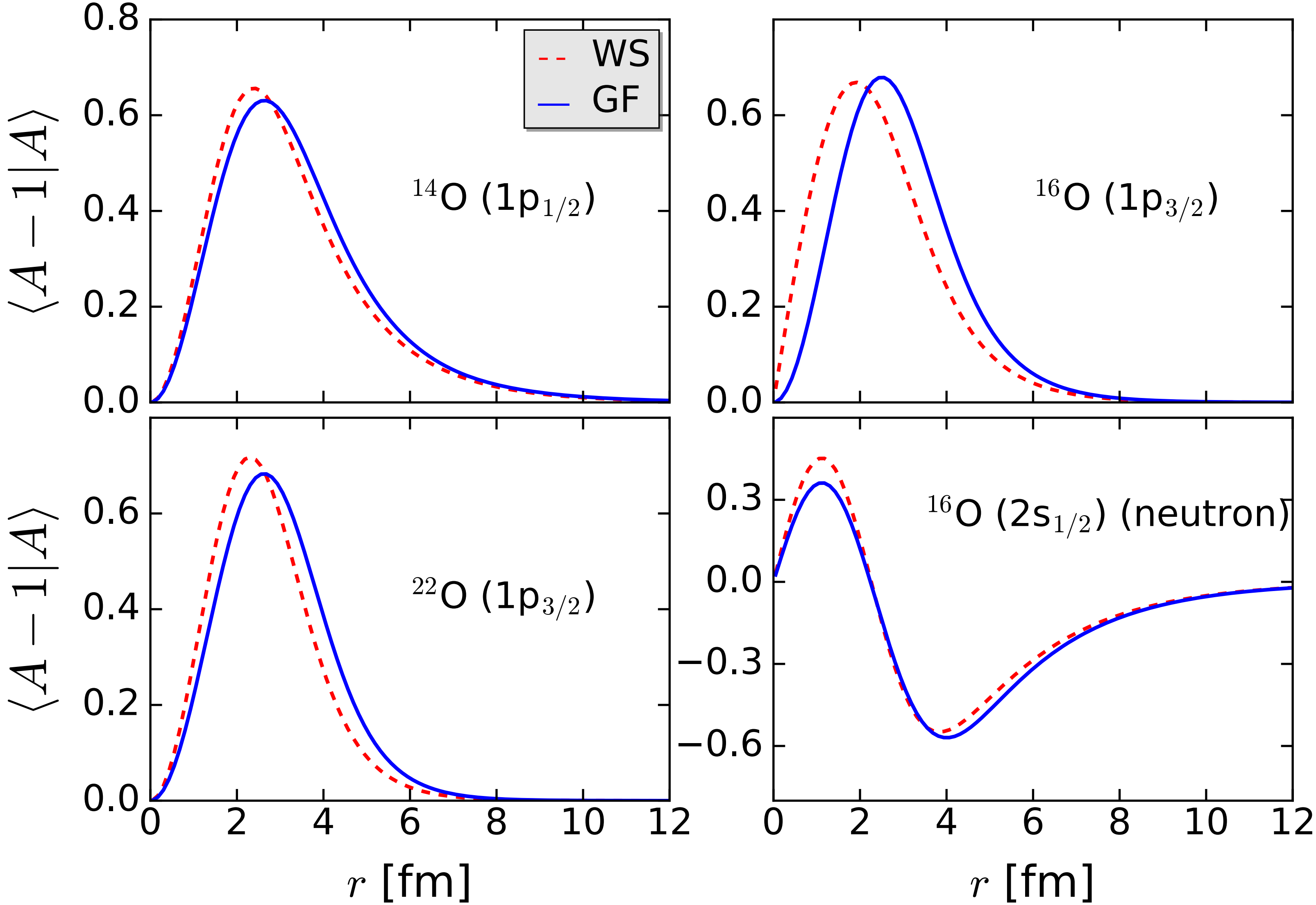
$$\frac{k^2}{2m}\psi_{l,j}(k) + \int dk' k'^2 \left(\Sigma^{l,j*}(k,k',E)\right)\psi_{l,j}(k') = \mathrm{E}\,\psi_{l,j}(k)$$

$$SF = \left|\left\langle \Phi_n^{(A-1)} \middle| \Phi_{g.s.}^A \right\rangle\right|^2$$ Calculated from overlap wavefunctions



open circles neutrons, closed protons

Overlap wavefunctions

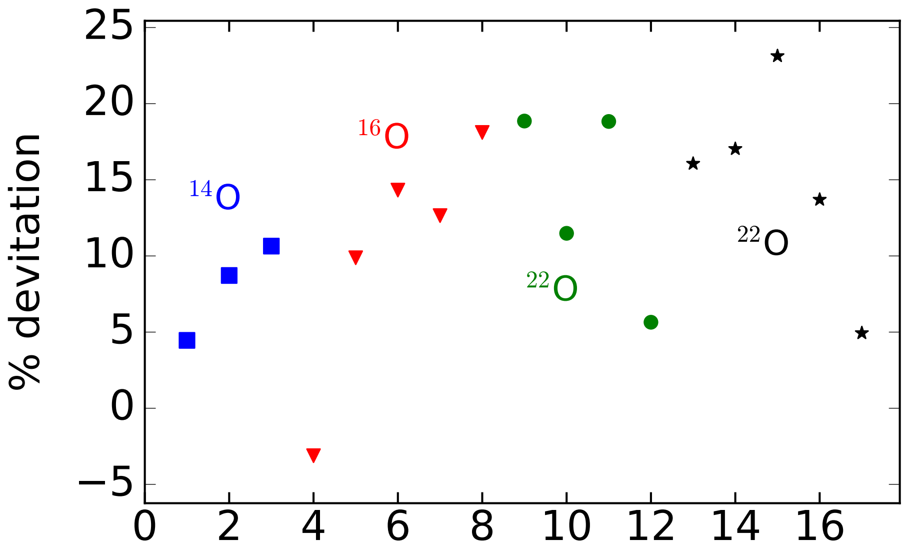


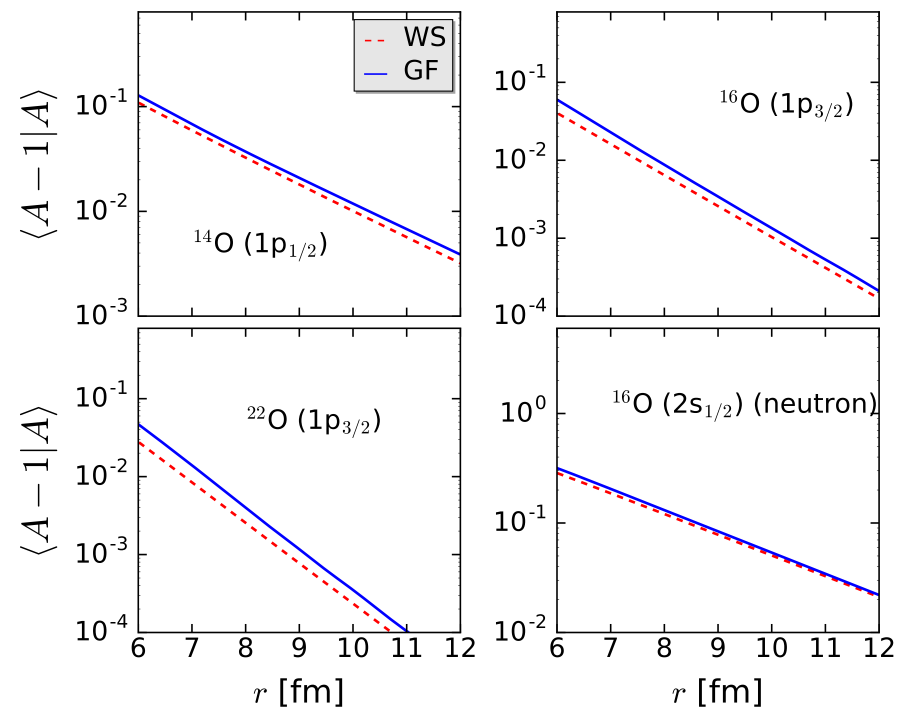
Collaboration with C. Bertulani

Nucleus (state)	E_B [MeV]	$\langle r^2\rangle^{1/2}_{WS}$ [fm]	$\langle r^2\rangle^{1/2}_{GF}$ [fm]	C_{WS} [$fm^{-1/2}$]	C_{GF} [$fm^{-1/2}$]	σ^{WS}_{qf} [mb]	σ^{GF}_{qf} [mb]	σ^{WS}_{kn} [mb]	σ^{GF}_{kn} [mb]	C^2S_{GF}
^{14}O ($\pi 1p_{3/2}$)	8.877	2.856	2.961	6.785	7.172	27.38	28.60	27.19	27.42	0.548

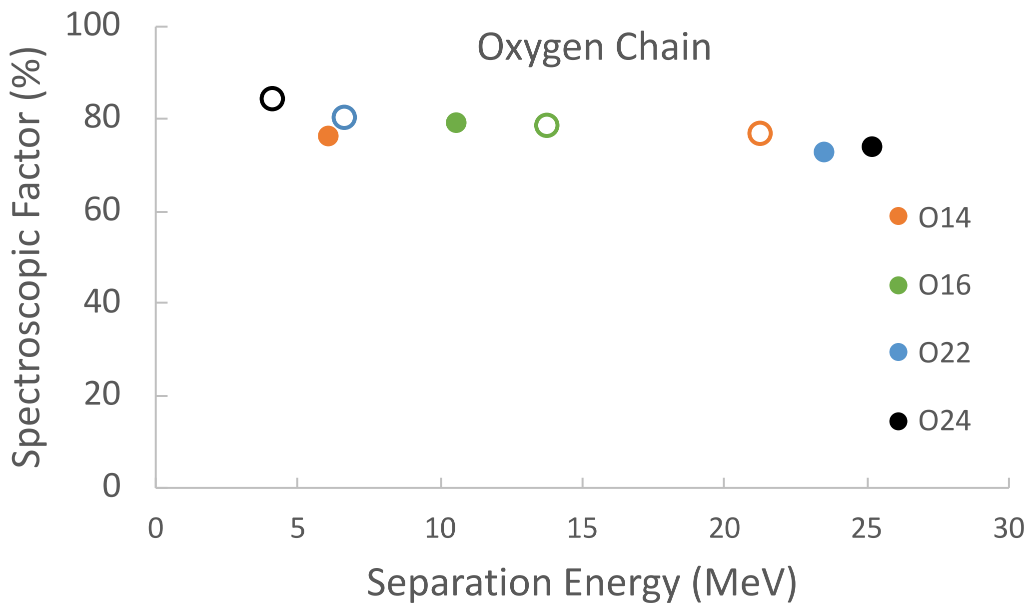
5% <1%

Deviation of quasifree (p, pn)
cross section calculation
for different wavefunctions
$(\sigma_{GF} - \sigma_{WS})/\sigma_{WS}$





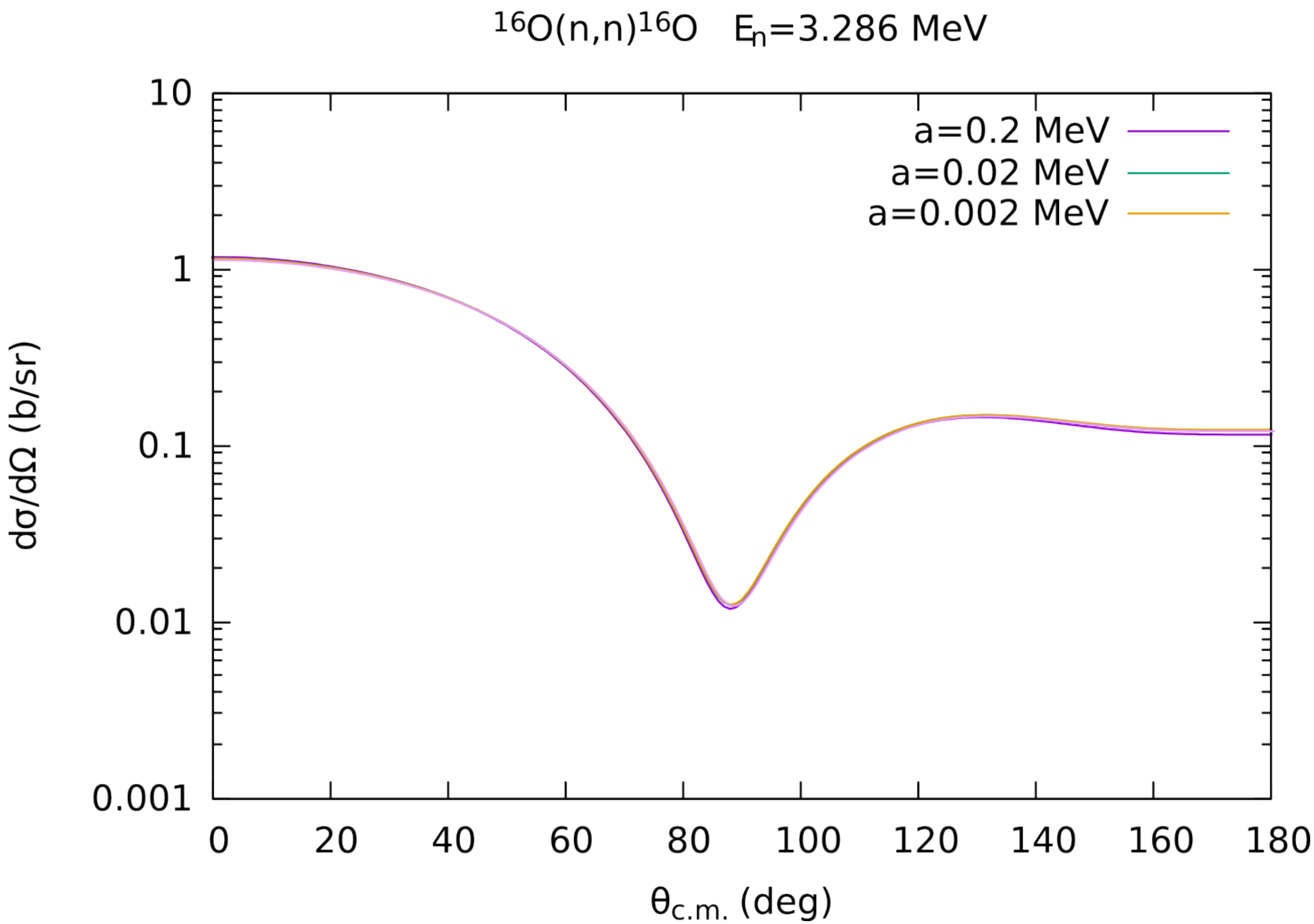
Collaboration with C. Bertulani

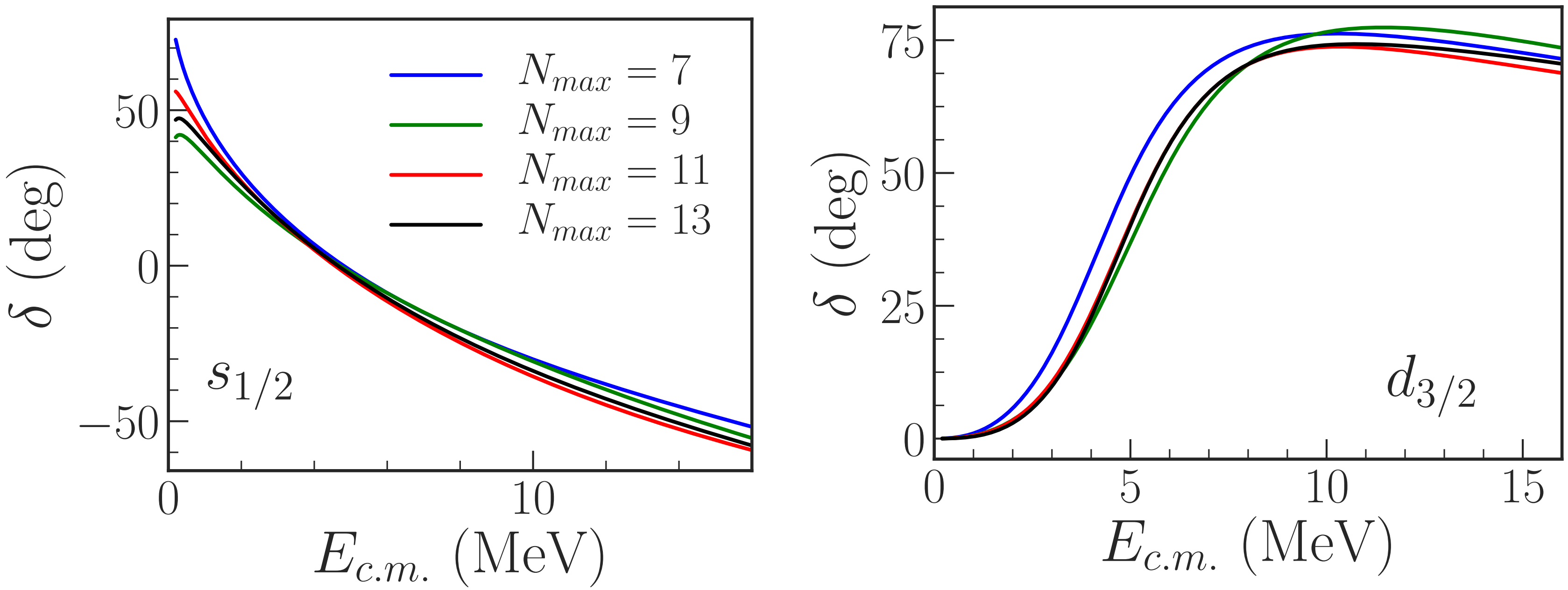

Oxygen Chain
Spectroscopic Factor (%)
100
80
60
40
20
0
0
5
10
15
20
25
30
Separation Energy (MeV)
O14
O16
O22
O24

«Imaginary» Parameter

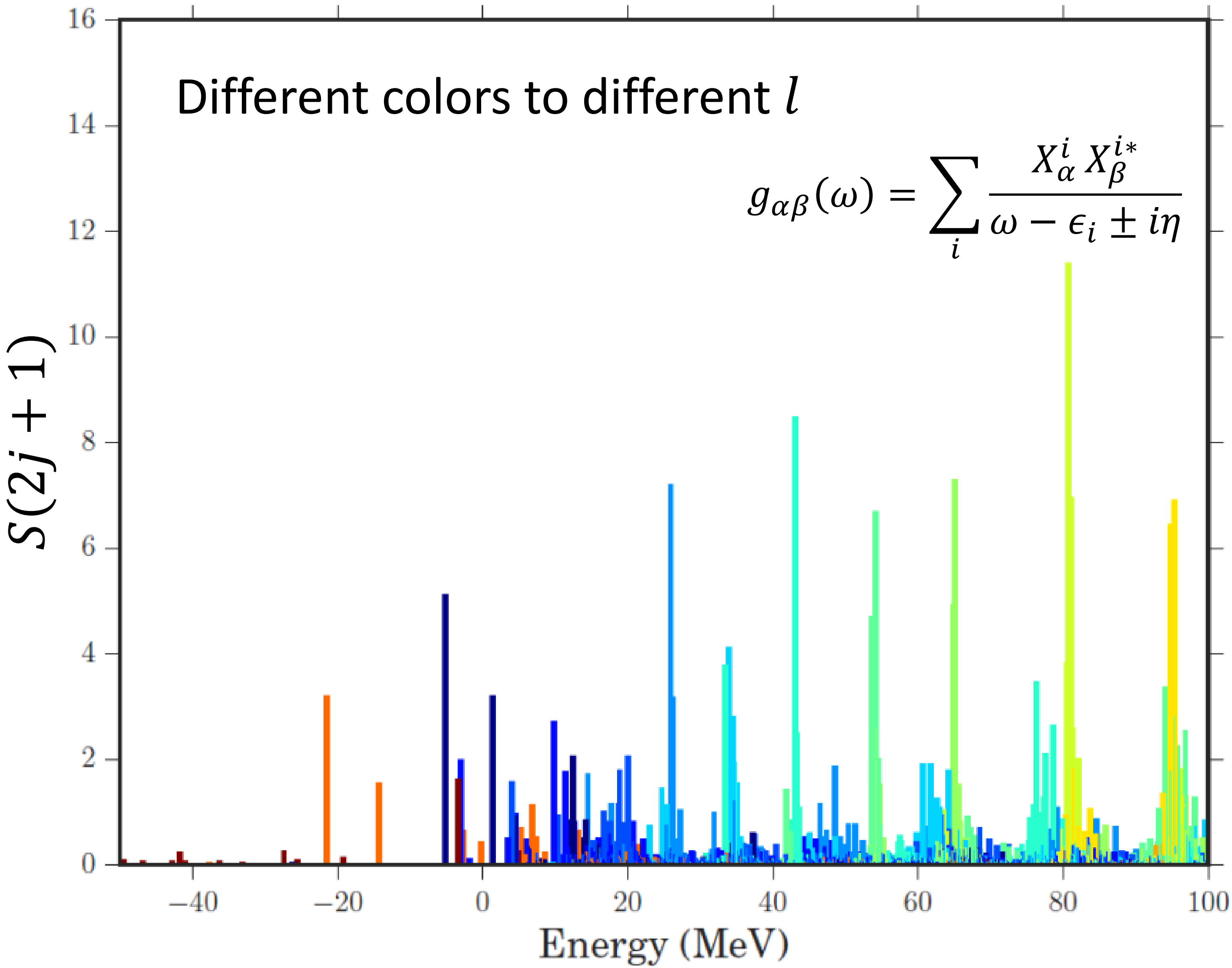
$$\Gamma(E) = \frac{1}{\pi} \frac{a\,(E - E_F)^2}{(E - E_F)^2 - b^2}$$

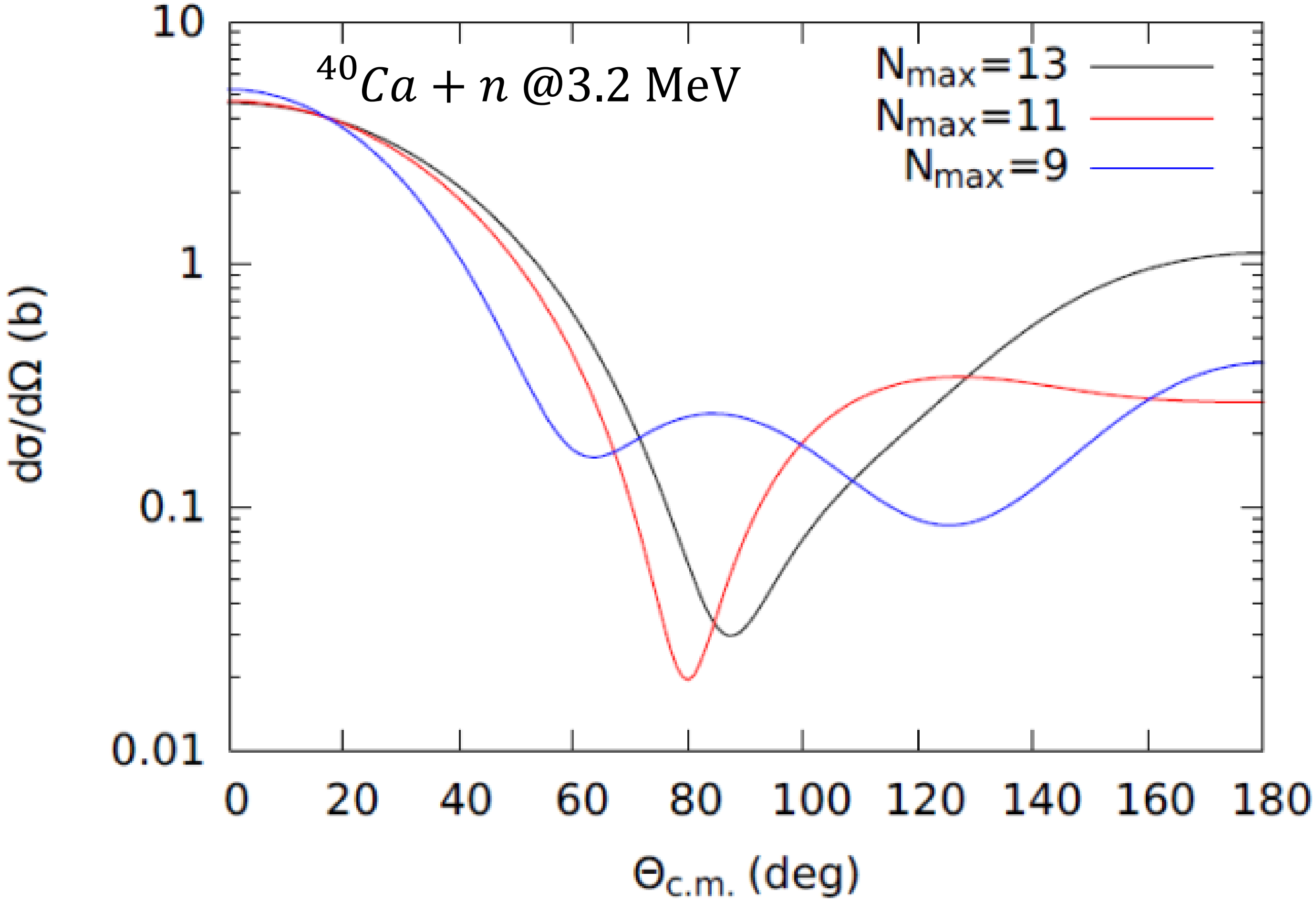
$b = 22.36$ MeV




$N_{max} = 7$
$N_{max} = 9$
$N_{max} = 11$
$N_{max} = 13$
$s_{1/2}$
δ (deg)
$E_{c.m.}$ (MeV)
$d_{3/2}$
δ (deg)
$E_{c.m.}$ (MeV)

^{16}O neutron propagator





$^{40}Ca + n$ @3.2 MeV
N_{max}=13
N_{max}=11
N_{max}=9
10
1
0.1
0.01
0
20
40
60
80
100
120
140
160
180
dσ/dΩ (b)
$\Theta_{c.m.}$ (deg)

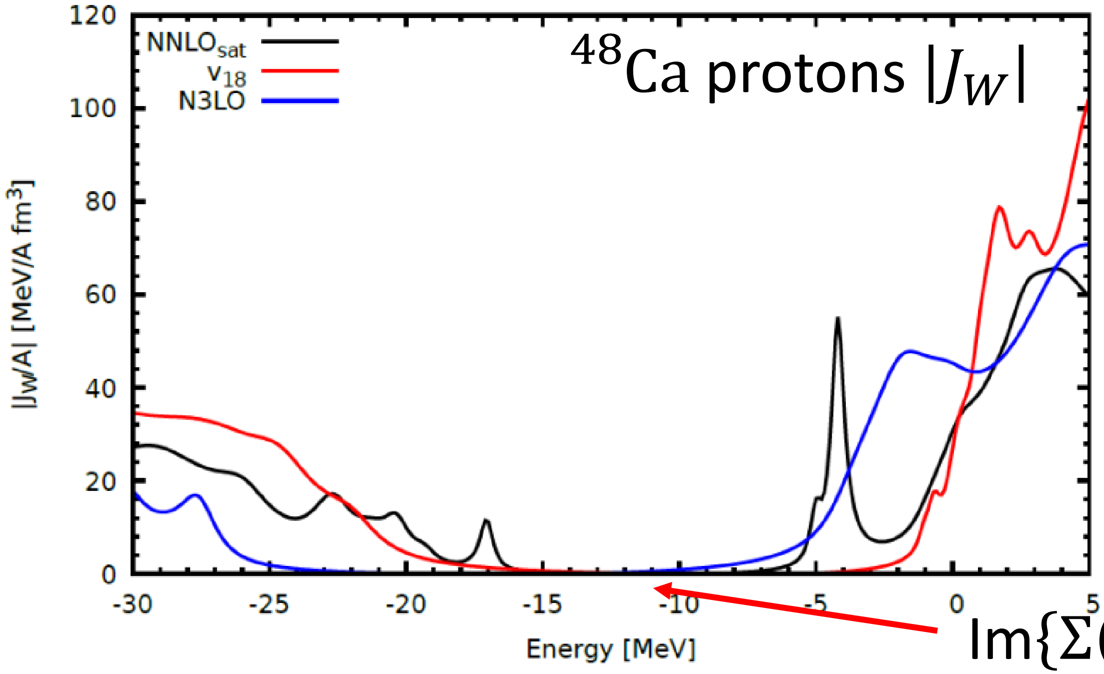
Volume integrals

$$J_W^{\ell}(E) = 4\pi \int drr^2 \int dr'r'^2 \mathrm{Im}\ \Sigma_0^{\ell}(r, r'; E)$$

Non local potential

$$J_V^{\ell}(E) = 4\pi \int drr^2 \int dr'r'^2 \mathrm{Re}\ \Sigma_0^{\ell}(r, r'; E).$$

$$\tilde{\Sigma}^{\ell j}_{n_a,n_b}(E) = \sum_r \frac{m^r_{n_a} m^r_{n_b}}{E - \varepsilon_r \pm i\eta}$$



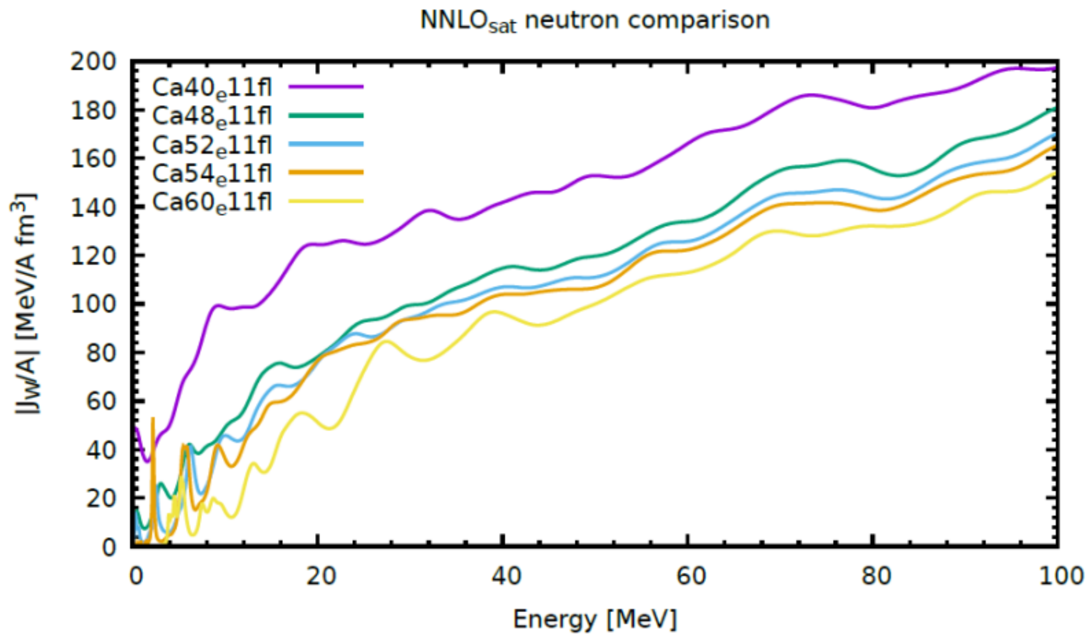
$\mathrm{Im}\{\Sigma(\epsilon_F)\} = 0$

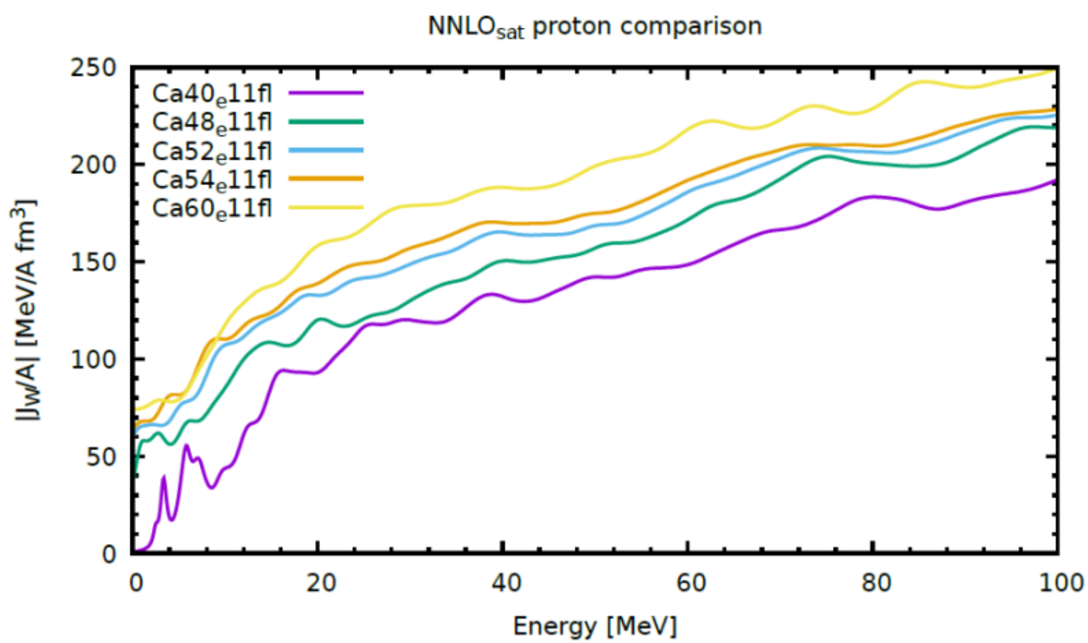
different Fermi energies and particle-hole gap for different interactions

S. Waldecker et al. PRC**84**, 034616(2011)

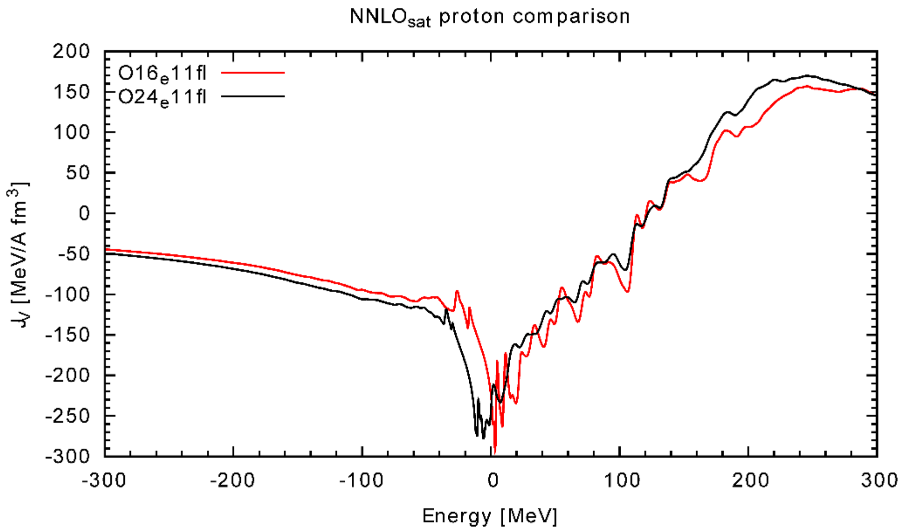
Ca isotopes

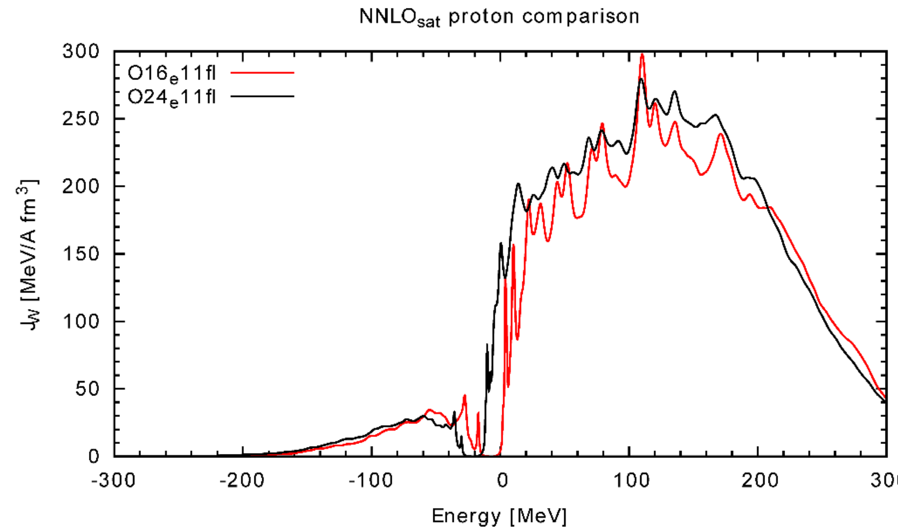
neutron and proton volume integrals of self energies.

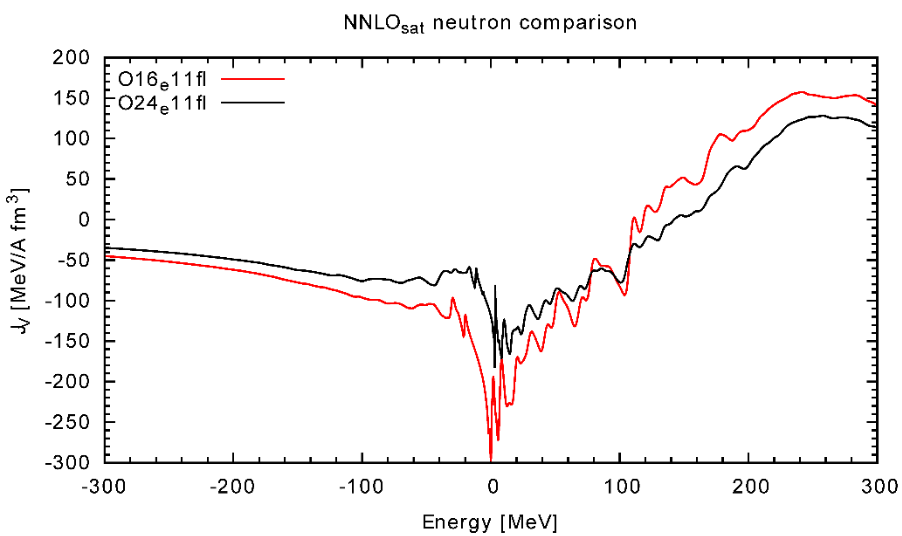


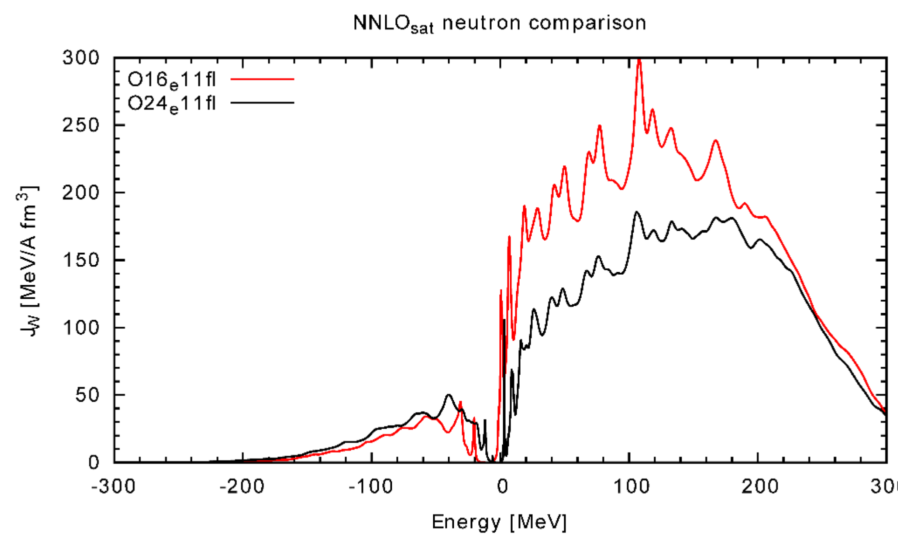


^{16}O and ^{24}O


NNLO$_{sat}$ proton comparison
O16$_e$11fl
O24$_e$11fl
J_V [MeV/A fm^3]
Energy [MeV]


NNLO$_{sat}$ proton comparison
O16$_e$11fl
O24$_e$11fl
J_W [MeV/A fm^3]
Energy [MeV]


NNLO$_{sat}$ neutron comparison
O16$_e$11fl
O24$_e$11fl
J_V [MeV/A fm^3]
Energy [MeV]


NNLO$_{sat}$ neutron comparison
O16$_e$11fl
O24$_e$11fl
J_W [MeV/A fm^3]
Energy [MeV]