A nonlocal (d,p) reaction story



told by Natalia Timofeyuk

from University of Surrey

on 12th day of May 2020

to all who came to listen



Asymmetry in SF reduction as seen in knockout reactions:

A. Gade et al, Phys. Rev. C 77, 044306 (2008)



Prologue

In the first decade of 2000s it was discovered that spectroscopic factors are reduced with respect to the shell model predictions. But those who did knockout and those who did transfer reactions disagreed if this reduction was the same or different for removal of strongly and weakly-bound nucleons...

No asymmetry in SF reduction is seen in (d,p) and (p,d) reactions Jenny Lee et al, Phys. Rev. C 73, 044608 (2006)



PHYSICAL REVIEW C 73, 044608 (2006)

Jenny Lee et al

II. METHODOLOGY

We thus propose the following consistent three-body analysis of ground-state-to-ground-state neutron transfer reaction data, taking HF theoretical input. We calculate the transfer reaction amplitudes using the Johnson-Soper (JS) adiabatic approximation to the neutron, proton, and target three-body system [13]. By this means we include the effects of the breakup of the deuteron in the field of the target and of the transfer of the neutron into (or out of) the breakup continuum.

...neutron-proton interaction [17]. <u>Nonlocality corrections, with</u> range parameters of 0.85 and 0.54 fm [18], were included in the proton and <u>deuteron channels</u>, respectively...

Where did the deuteron nonlocality range of 0.54 fm come from?

Prof. Ron Johnson:

"The nonlocality for deuterons is not 0.54 but 0.45 fm! We have shown in our paper that $\beta_d \approx \beta_N/2$ ".

R. C. Johnson and P. J. R. Soper, <u>"Relation between the Deuteron and Nucleon</u> <u>Optical Potentials</u>" Nucl. Phys. A182, 619 (1972) Watanabe model with Perey-Buck nonlocal optical potentials

$$\int d\mathbf{r}' V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')$$

Optical model with nonlocal potentials

Origin of nonlocality:

Complex structure of target A. We ignore target excitations and assume that they are all included in optical potentials.

Result: incoming nucleon may disappear from elastic channel but reappear somewhere else.

<u>Perey-Buck</u> form of nonlocal potentials

$$V(\mathbf{r},\mathbf{r'}) = U_N\left(\frac{|\mathbf{r}+\mathbf{r'}|}{2}\right) \frac{\exp\left(-(\mathbf{r}-\mathbf{r'})^2/\beta^2\right)}{\pi^{3/2}\beta^3}$$

$\beta \approx 0.85 \, fm$ is non-locality range

F. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

Leading order:

$$\left(T + U_{loc}^{0}(r) - E\right)\varphi(r) = 0$$
$$U_{loc}^{0} = U_{N} \exp\left[-\frac{\mu\beta^{2}}{2\hbar^{2}}\left(E - U_{loc}^{0}\right)\right]$$

Next-to-leading order: H. Fiedeldey, Nucl. Phys. 77, 149 (1966) $\left(T + U_{loc}^{0}(r) + \Delta U - E\right) \varphi(r) = 0$

$$\psi(\mathbf{r}) \approx \varphi(\mathbf{r}) \exp\left[\frac{\mu\beta^2}{4\hbar^2} U_{loc}^0(\mathbf{r})\right]$$
1.0
0.8
0.8

Local-equivalent models have been developed for

- two-body scattering
- Watanabe model
- Does a local-equivalent model exists for ADWA? (Adiabatic Distorted Wave approximation)
- Is using the Perey factor in ADWA calculations legitimate?

Exact nonlocal Faddeev (d,p) calculations with Perey-Buck optical potentials differ from those using local-equivalents.

A. Deltuva, Phys. Rev. C 79, 021602 (2009).



Chapter I

Which explains how three-body problem with energyindependent optical potentials should be solved in the ADWA, what unusual properties this problem has and suggests some ways to go beyond the adiabatic approximation.



$$T_{(d,p)} = \left\langle \chi_{pB}^{(-)} \phi_p \phi_B \middle| V_{np}(r) \middle| \Psi(r,R) \phi_A \right\rangle$$

<u>ADWA</u>: $\Psi(r,R) \approx \phi_d(r)\psi(R)$

 $\left(T_R + U_{dA}^{ADWA}(\boldsymbol{R}) + V_C - E_d\right)\psi(\boldsymbol{R}) = 0$

 $U_{dA}^{ADWA}(\mathbf{R}) = -\int d\mathbf{r} \,\phi_1(\mathbf{r})(U_{nA} + U_{pA})\phi_d(\mathbf{r})$

R. C. Johnson and P.C. Tandy, Nucl. Phys. A235 56 (1974)

ADWA model with nonlocal p-A and n-A potentials

N.K. Timofeyuk and R.C. Johnson, Phys. Rev. C 87, 064610 (2013)

$$(T_R + V_C - E_d) \psi(\mathbf{R}) = -\int d\mathbf{R}' U_{dA}^{ADWA}(\mathbf{R}, \mathbf{R}') \psi(\mathbf{R}')$$

$$U_{dA}^{ADWA}(\mathbf{R}, \mathbf{R}') = \int d\mathbf{r} \, \phi_1(\mathbf{r}) \left[V_{nA} \left(2\mathbf{R}' - \mathbf{R} + \frac{\mathbf{r}}{2}, \mathbf{R} + \frac{\mathbf{r}}{2} \right) \phi_d(\mathbf{r} + 2(\mathbf{R}' - \mathbf{R})) \right]$$

 $\phi_1(\mathbf{r}) = \frac{V_{np}\phi_d(\mathbf{r})}{\langle \phi_d | V_{np} | \phi_d \rangle}$

If we replace $\phi_1(\mathbf{r})$ by $\phi_d(\mathbf{r})$ we will end up with nonlocal Watanabe model with Perey-Buck nonlocal optical potentials as in R. C. Johnson and P. J. R. Soper, Nucl. Phys. A182, 619 (1972)

Leading-order solution:

 $d\mathbf{R}' U_{dA}^{ADWA}(\mathbf{R}, \mathbf{R}')\psi(\mathbf{R}') \approx U_{loc}^{0}(\mathbf{R})\psi(\mathbf{R})$

$$U_{dA}^{\text{loc}} \neq M_0 U_{dA} \exp\left[-\frac{\mu_d \beta_d^2}{2\hbar^2} \left(E_d - U_{dA}^{\text{loc}} - U_C\right)\right]$$
$$U_{dA} = U_{pA} + U_{nA}$$

This equation is to be solved at each R

Comparison between exact ADWA calculations and LO and NLO models.

G.W. Bailey, N.K.Timofeyuk and J.A.Tostevin, Phys. Rev. C 95, 124603 (2017)

Conclusion: a local-equivalent ADWA model exists

Next-to-leading-order solution:

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 $U_{dA}^{\rm loc} \rightarrow U_{dA}^{\rm loc} + \Delta U$

 $\psi_{dA}(R) \rightarrow P(R) \psi_{dA}(R)$



Is the local-equivalent ADWA model the same as used in widely-used reaction codes?

$$U_{dA}^{\text{loc}} = M_0 U_{dA} \exp\left[-\frac{\mu_d \beta_d^2}{2\hbar^2} \left(E_d - U_{dA}^{\text{loc}} - U_C\right)\right]$$

Johnson and Soper had $\mathcal{M}_0=1$ and they have shown that

 $U_{dA}^{\rm loc}(E_d) = U_{pA}^{\rm loc}(E_d/2) + U_{nA}^{\rm loc}(E_d/2)$

But in ADWA $M_0 \approx 0.8$ for deuteron Hulthén wave function. It is related to n-p kinetic energy within the range of the short-range n-p interaction.

N.K. Timofeyuk and R.C. Johnson, Phys. Rev. C 87, 064610 (2013)

For $\mathcal{N} = \mathbb{Z}$ nuclei a solution for the effective local d-A potential exists:

$$U_{loc}^{0}(E_d) = \frac{1}{\alpha_2} \left(\frac{\beta}{2\beta_d}\right)^2 \left(U_{nA}^{loc}(E_n) + U_{pA}^{loc}(E_p)\right)$$

where

$$E_n = \frac{\alpha_2}{2} \left(\frac{2\beta_d}{\beta}\right)^2 \left(E_d - \bar{U}_C\right) + E_0$$

$$E_p = \frac{\alpha_2}{2} \left(\frac{2\beta_d}{\beta}\right)^2 \left(E_d - \bar{U}_C\right) + E_0 + \bar{V}_{coul}$$

 $E_0 \approx 40 \text{ MeV}$ is related to M_0 and has a meaning of some additional energy.

U_{dA}^{loc} versus Johnson-Soper potentials and (d,p) cross sections



More on nonlocal versus local (d,p) calculations can be found in L.J.Titus, F.M.Nunes, G.Potel, Phys.Rev. C 93, 014604 (2016) A.Ross, L.J.Titus, F.M.Nunes, Phys.Rev. C 94, 014607 (2016)

$U_{dA}^{\text{loc}} = M_0 U_{dA} \exp\left[-\frac{\mu_d \beta_d^2}{2\hbar^2} \left(E_d - U_{dA}^{\text{loc}} - U_C\right)\right]$ $M_0 \text{ is determined by} \quad \left\langle T_{np} \right\rangle_V = \frac{\left\langle \phi_d | T_{np} V_{np} | \phi_d \right\rangle}{\left\langle \phi_d | V_{np} | \phi_d \right\rangle}$

Sensitivity to the NN –potential model

NN potential	M ₀	$\left\langle T_{np}\right\rangle_{V}$ (MeV)	E_0 (MeV)	β_d (fm)
Hulthén	0.761	106.6	35.5	0.451
Reid	0.526	245.8	79.3	0.426
AV18	0.561	218.0	70.7	0.432
CD-Bonn	0.719	112.5	41.4	0.449
χEFT N4LO (0.8)	0.529	247.2	74.7	0.436
χEFT N4LO (0.9)	0.577	190.1	66.3	0.437
χEFT N4LO (1.0)	0.623	154.6	58.6	0.439
χEFT N4LO (1.1)	0.668	122.6	50.9	0.442
χEFT N4LO (1.2)	0.711	88.2	43.9	0.445

 M_0 is sensitive to the NN-model mainly through the deuteron d-state.





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 $^{26}Al(d,p)^{27}Al at E_d = 12 MeV$

Strong sensitivity to the choice of the deuteron wave function model



G.W. Bailey, N.K. Timofeyuk and J.A. Tostevin, Phys. Rev. Lett. 117, 162502 (2016)

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Beyond the ADWA: CDCC calculations with nonlocal optical potentials

Local-equivalent CDCC model:

M. Gómez-Ramos and N.K. Timofeyuk, Phys. Rev. C 98 011601(R) (2018)

LECDCC

$$(T_R + U_C(R) - E_d)\chi_i(\boldsymbol{R}) = -\sum_{i'} U_{ii'}^{\text{loc}}(\boldsymbol{R})\chi_{i'}(\boldsymbol{R})$$

 U_{ii}^{loc} are found from solving a coupled transcendental matrix equations

Sensitivity to the choice of the NN model is reduced beyond the adiabatic approximation.

Weak sensitivity of three-body (d, p) reactions to n-p force models has been also confirmed by exact Faddeev calculations A. Deltuva, Phys. Rev. C 98, 021603(R) (2018)



Another view of treating nonlocality

Local-equivalent N-A optical model in the next-to-leading order has velocity-dependent optical potential:

$$(T - E) \psi(\mathbf{r}) = -\int d\mathbf{r}' V(\mathbf{r}, \mathbf{r}') \psi(\mathbf{r}')$$
$$\approx -\widetilde{U}_{loc}(\mathbf{r}) \psi(\mathbf{r}) - \nabla F(\mathbf{r}) \nabla \psi(\mathbf{r})$$

 $\widetilde{U}_{loc}(r) = U^0_{loc} + \Delta U$

Three-body model with velocity-dependent potentials:

 $\left(T_3 + V_{np}(\mathbf{r}) + \widetilde{U}_{loc}^{nA}(r_n) + \nabla_n F(r_n)\nabla_n\right)$

Perey-factorization of the 3-body wave function

 $\Psi(\boldsymbol{R},\boldsymbol{r}) = P_n(\boldsymbol{r}_n)P_p(\boldsymbol{r}_p)\varphi(\boldsymbol{R},\boldsymbol{r})$

leads to the local 3-body equation

 $(T_3 + V_{np}(\mathbf{r}) + \widetilde{U}_{nA}^{\text{eff}}(r_p) + \widetilde{U}_{nA}^{\text{eff}}(r_p)$

If this equation is solved using adiabatic approximation then it leads a model very closed to widely used local ADWA with correction for nonlocality! ()

N.K.Timofeyuk, J.Phys. G46, 065103 (2019)

Solving 3-body problem with velocity-dependent potentials: $(T_3 + V_{np}(\mathbf{r}) + \tilde{U}_{nA}^{eff}(r_p) + \tilde{U}_{nA}^{eff}(r_p) + \text{small}(\mathbf{R}, \mathbf{r}) - E)\varphi(\mathbf{R}, \mathbf{r}) = 0$ in the CDCC and applying Perey factors $\Psi(\mathbf{R}, \mathbf{r}) = P_n(\mathbf{r}_n)P_p(\mathbf{r}_p)\varphi(\mathbf{R}, \mathbf{r})$

M. Gómez-Ramos and N.K. Timofeyuk, J. Phys. G46, 085102 (2019)



Exact nonlocal Faddeev calculations from A. Deltuva, Phys. Rev. C 79, 021602 (2009) are compared to the CDCC in Prog. Part. Nucl. Phys. 111, 103738 (2020)



Which aims to understand how a three-body problem should be formulated when nonlocal nucleon optical potentials are explicitly energy-dependent and proposes an approximation for a three-body problem consistent with the ADWA.

Local-equivalent potential is always energy-dependent

$$U_{loc}^{0} = U_{N} \exp\left[-\frac{\mu\beta^{2}}{2\hbar^{2}}\left(E - U_{loc}^{0}\right)\right]$$

Is this energy-dependence the same as the one of phenomenological optical potentials widely-used in (d,p) calculations? Chapel-Hill optical potential systematics:

 U_N from U_{loc}^0 for $\mathcal{E} = 5$ to 50 MeV



The energy-dependence of the phenomenological imaginary part differs from that arising from energy-independent nonlocal potential.

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Two-body nucleon scattering of complex nuclei :

A N

 χ_0 is found from the two-body equation:

$$(T+V_{opt}-E)\chi_0=0$$

$$\Psi = \phi_{g.s.} \chi_0(\mathbf{r}) + \sum_{i \neq 0} \phi_i \chi_i(\mathbf{r})$$
$$\Psi_P = P \Psi \qquad \Psi_Q = Q \Psi$$
$$Q = \sum_{i \neq 0} |\phi_i\rangle \langle \phi_i|$$

Operator Q projects the wave function into all excited states.

$$V_{opt} = \left\langle \phi_{g.s.} \middle| v_{NA} + v_{NA}Q \frac{1}{E_N - Qv_{NA}Q}Q \middle| \phi_{g.s.} \right\rangle$$
$$v_{NA} = \sum_{i=1}^{A} v_{Ni}$$

All target excitations are hidden into energy-dependent on non-local non-hermitian optical potential.

Optical potentials in the A + n + p three-body model



$$\Psi = \phi_{g.s.} \chi_0(\boldsymbol{r}, \boldsymbol{R}) + \sum_{i \neq 0} \phi_i \chi_i(\boldsymbol{r}, \boldsymbol{R})$$
$$\Psi_P = P \Psi \qquad \Psi_Q = Q \Psi$$
$$Q = \sum_{i \neq 0} |\phi_i\rangle \langle \phi_i|$$

Ground-state channel function can be found from three-body model

$$(T_3 + V_{np} + \left\langle \phi_{g.s.} \middle| V_{opt} \middle| \phi_{g.s.} \right\rangle - E_3) \chi_0 = 0$$

with the optical potential

$$V_{opt} = U_{nA} + U_{pA} + U_{nA} \frac{Q}{e} U_{pA} + U_{pA} \frac{Q}{e} U_{nA} + \dots$$
$$U_{NA} = v_{NA} + v_{NA} \frac{Q}{e} U_{NA}$$

Optical potential for 3-body system has two-body and three-body terms

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)

Two-body force in a three-body system

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)



Neglecting multiple scattering terms gives us the Schrödinger equation

$$(T_{3} + V_{np} + \left\langle \varphi_{g.s.} \middle| U_{nA} + U_{pA} \middle| \varphi_{g.s.} \right\rangle - E_{3})\chi_{0} = 0$$

Comparing N-A operators:

in A+*n*+*p system*:

$$U_{NA} = v_{NA} + v_{NA} \frac{Q}{E_3 + i0 - T_3 - V_{np} - (H_A - E_A)} U_{NA}$$

in N+A system:

$$U_{NA} = v_{NA} + v_{NA} \frac{Q}{E_N + i0 - T_{NA} - (H_A - E_A)} U_{NA}$$

Two-body force in three-body system differs from two-body optical potential! Dealing with operators U_{nA} and U_{pA} in ADWA

$$\int (T_R + \langle \varphi_1 \varphi_A | U_{nA} + U_{pA} | \varphi_d \varphi_A \rangle - E_d) \chi_{dA}^{(+)}(\mathbf{R}) = 0$$

Averaging procedure gives

$$\langle \varphi_{1} \varphi_{A} | U_{NA} | \varphi_{d} \varphi_{A} \rangle$$

$$\approx \left\langle \varphi_{A} \middle| v_{NA} + v_{NA} \frac{Q}{E_{\text{eff}} + i0 - T_{N} - (H_{A} - E_{A})} U_{NA} \middle| \varphi_{A} \right\rangle$$

where

$$E_{eff} = \frac{1}{2}E_d + \frac{1}{2}\frac{\left\langle \varphi_d \left| V_{np} T_{np} \right| \varphi_d \right\rangle}{\left\langle \varphi_d \left| V_{np} \right| \varphi_d \right\rangle}$$

half the n-p kinetic energy in deuteron ranges between 44 and 120 MeV Comparing to the N-A optical potential:

$$\langle \varphi_A | U_{NA} | \varphi_A \rangle \approx \left\langle \varphi_A \left| v_{NA} + v_{NA} \frac{Q}{E_N + i0 - T_N - (H_A - E_A)} U_{NA} \right| \varphi_A \right\rangle$$

Three-body problem for (d,p) reactions should be solved with energyindependent nonlocal nucleon potentials taken at effective energy equal to half the deuteron energy plus a shift.



Including multiple scattering effects in the leading order within the ADWA

$$V_{opt} = U_{nA} + U_{pA} + U_{nA} \frac{Q}{e} U_{pA} + U_{pA} \frac{Q}{e} U_{nA} + \dots$$

$$U^{(0)} \qquad U^{(1)}$$

$$\langle \varphi_1 \varphi_A | U^{(0)} + U^{(1)} | \varphi_d \varphi_A \rangle$$

$$\approx \sum_{N=n,p} \left\langle \varphi_1 \varphi_A | v_{NA} + 2v_{NA} \frac{Q}{E_{\text{eff}} + i0 - T_N - Qv_{NA}Q} v_{NA} Q | \varphi_d \varphi_A \right\rangle$$

$$\begin{split} &\langle \varphi_1 \varphi_A \left| U^{(0)} + U^{(1)} \right| \varphi_d \varphi_A \rangle \\ &\approx 2 \langle \varphi_1 \varphi_A \left| U^{(0)} \right| \varphi_d \varphi_A \rangle - \sum_{N=n,p} \langle \varphi_1 \varphi_A | v_{NA} | \varphi_d \varphi_A \rangle \end{split}$$

M.J.Dinmore, N.K.Timofeyuk, J.S.Al-Khalili, R.C.Johnson Phys.Rev. C 99, 064612 (2019) *Relation to phenomenological optical potentials*

$$V_{NA}^{\text{opt}}(E) = V_{NA}^{HF} + \Delta V_{NA}^{\text{dyn}}(E)$$

$$\Delta V_{NA}^{\rm dyn}(E) = iW_{NA}(E) + \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}$$

$$\langle \phi_A | U^{(0)} + U^{(1)} | \phi_A \rangle$$

= $V_{nA}^{HF} + 2\Delta V_{nA}^{\text{dyn}}(E) + V_{pA}^{HF} + 2\Delta V_{pA}^{\text{dyn}}(E)$

Dynamical part of the phenomenological optical potential, taken at a shifted energy, should be doubled. Nonlocal Dispersive Optical Model (NLDOM) has real dynamical part

Hulthén deuteron wave function corresponding to $E_{\rm eff} = 57 \; {\rm MeV}$

NLDOM spectroscopic factor is 0.73



M.J.Dinmore et al, Phys.Rev. C 99, 064612 (2019)

Giannini-Ricco-Zucchiati (GRZ) global optical potential (energy-dependent)



Giannini-Ricco (GR) global optical potential (energyindependent)





Nonlocal Dispersive Optical Model (NLDOM) has real dynamical part

Hulthen deuteron wave function corresponding to $E_{eff} = 57 \text{ MeV}$ Giannini-Ricco-Zucchiati (GRZ) global optical potential (energy-dependent)

Giannini-Ricco (GR) global optical potential (energyindependent)

Both don't have real dynamical part



Conclusions:

- N-A interactions in three-body systems are not the same as N-A optical potentials
- Induced three-body effects are important

What is missing?

- Other multiple scattering terms
- Non-adiabatic effects

Outstanding problem: How to use optical potentials in three-body systems? How to link properly many- and few-body degrees of freedom?

Why is it important?

- Phenomenological systematics of nonlocal optical potentials are being established
- Ab-initio developments of optical potentials
- Three-body description of breakup, knockout reactions, Coulomb excitations
- Three-body bound states

Why does phenomenological description of three-body reactions and bound states without three-body forces work? More on interplay between three- and many-body degrees of freedom can be found in

Progress in Particle and Nuclear Physics 111 (2020) 103738



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Review

Theory of deuteron stripping and pick-up reactions for nuclear structure studies



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Epilogue

Some widely-used reactions codes have an option to use correction for nonlocality of optical potentials. This option has not been justified for many years. Now we find out that such a procedure comes from local-equivalent potentials with addition of velocity-dependent forces, which is equivalent to the next-to-leading order treatment of nonlocality.

However, such widely-used corrections for nonlocality are very different from what exact adiabatic treatment of (d,p) reactions predicts.

Adiabatic treatment of (d,p) reactions with nonlocal potentials suffers from artefact associated with high n-p momenta in deuteron. When nonlocal potentials are involved (d,p) reactions should be treated beyond adiabatic approximation.

Attempting to understand how energy-dependence of nonlocal optical potentials should be treated within (d,p) reaction theory lead to realisation that n-A and p-A forces in d+A system are not the same as nucleon optical potentials. They depend on the position of and interaction with third particle and multiple scattering within the n+p+A system. Adiabatic description of this problem has been work out.

But how to treat p-A and n-A interactions in p+n+A system beyond adiabatic approximation and why phenomenological three-body descriptions of three-body reactions without 3-body force are successful? p n deuteron n

To be continued some time in the future....

Thanks to everyone who stayed and listened

The end

Nonlocal v local-equivalent [Prog. Part. Nucl. Phys. 111 103738 (2020)]



NLDOM: nonlocal v local-equivalent [Phys. Rev. C94, 034609 (2016)]



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No sensitivity to NN model when nucleon optical potentials are local:

0



Perey-effect in CDCC

Local-equivalent nucleon potential + first-order correction

CDCC is applied to $\varphi(\mathbf{R},\mathbf{r})$

M. Gómez-Ramos and N.K. Timofeyuk, J. Phys. G46, 085102 (2019)

