Analysis of ^{10}Li and ^{11}Li through intermediate-energy (p, pn) and low-energy transfer reactions

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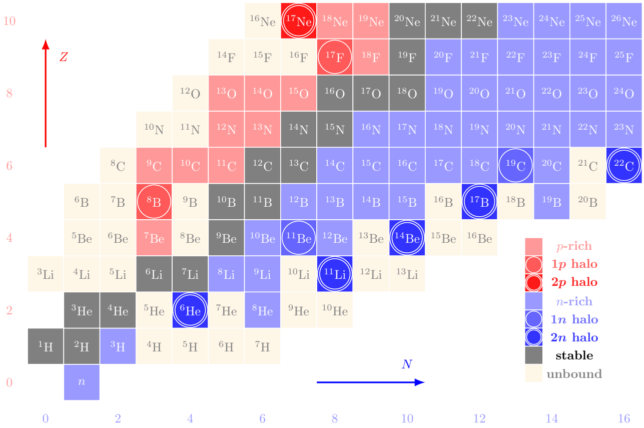
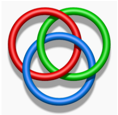
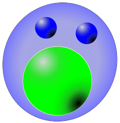
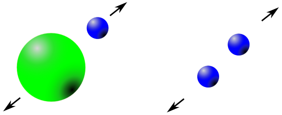
Borromean nuclei









- 3-body system bound, but 2-body subsystems unbound
- Close to neutron (proton) driplines
- Halo structure

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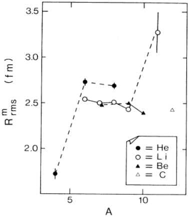
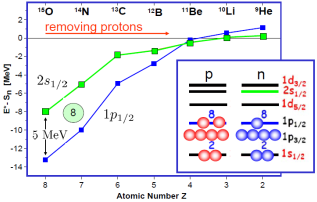
^{11}Li, ^{10}Li

- Halo structure



I. Tanihata *et al*, PRL **55**, 2676 (1985)

- Parity inversion



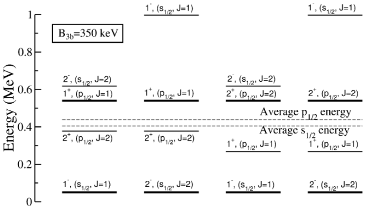
From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

- Unclear Structure

Table 2
Results of various experiments for low-lying states of ^{10}Li (Energies and widths in MeV).

Year	Reaction	E_r	Γ	ℓ	Ref.
1997	^{10}Be(^{12}C, ^{12}N)	0.24(4)	0.10(7)		[10]
1999	^{9}Be(^{9}Be, ^{8}B)	0.50(6)	0.40(6)		[8]
1999	fragmentation	<0.05		*s*	[9]
2001	*p* removal from ^{11}Be			g.s. is *s*	[11]
2003	^{9}Li(*d*, *p*)	0.35(11)	<0.32		[7]
		or <0.2	–		
		plus 0.77(24)	<0.62		
2006	^{9}Li(*d*, *p*)	~0		*s*	[12]
		~0.38	~0.2	*p*	
2015	2*p* removal from ^{12}B	0.11(4)	0.2		[13]
		0.50(10)	0.8	both *p*	

H.T. Fortune PLB 760, 577 (2016)



E. Garrido *et al* NPA 700, 117 (2002)

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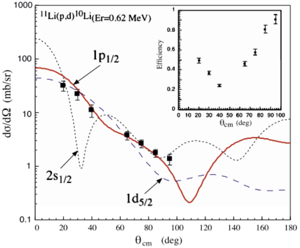
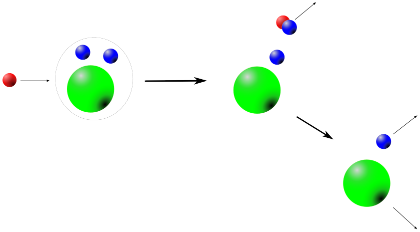
$p(^{11}\mathrm{Li}, d)^{10}\mathrm{Li}$ @ 5.7 MeV/A

- Exp. data
  (A. Sanetullaev *et al*
  PLB 755, 481 (2016))



- Participant-spectator model



- DWBA formalism

$$\mathcal{T}_{if} = \left\langle \chi_{d^{10}\mathrm{Li}} \varphi_d \phi_{^{10}\mathrm{Li}}(E_{^{10}Li}) | V_{pn} | \chi_{p^{11}\mathrm{Li}} \phi_{^{11}\mathrm{Li}} \right\rangle$$

- Spectator model (^{10}Li not modified by reaction)

$$\mathcal{T}_{if} \sim \left\langle \chi_{d^{10}\mathrm{Li}} \varphi_d | V_{pn} | \chi_{p^{11}\mathrm{Li}} \varphi_{lj}(E_{^{10}\mathrm{Li}}) \right\rangle$$

$$\varphi_{lj} = < \phi_{^{10}\mathrm{Li}}(E_{^{10}\mathrm{Li}}) | \phi_{^{11}\mathrm{Li}} >$$

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Overlap function $< \phi_{^{10}\text{Li}}(E_{^{10}\text{Li}}) | \phi_{^{11}\text{Li}} >$

$|^{11}\text{Li}\rangle$

- Only ground state needed ($B_{3b} = 396\text{keV}$)
- 3-body calculation ($^9\text{Li}+n+n$)
- $V_{n^9\text{Li}}$, V_{nn}, $V_{nn^9\text{Li}}$
- Expansion in a transformed harmonic oscillator (THO) basis with hyperspherical harmonic functions
- $J^\pi = 0^+$ (w/o ^{9}Li Spin)
  $J^\pi = 3/2^-$ (w/ ^{9}Li Spin)

$\langle ^{10}\text{Li}|$

- Unbound (all positive energies possible)
- $E_{^{10}\text{Li}} = 0 - 3$ MeV considered
- Scattering states for $V_{n^9\text{Li}}$
- $J^\pi = 1/2^+, 1/2^-$ (w/o ^{9}Li Spin)
  $J^\pi = 1^-, 2^-, 1^+, 2^+$ (w/ ^{9}Li Spin)

	E_r (MeV)		a (fm)		$\%p_{1/2}$	$\%s_{1/2}$	r_{mat} (fm)	r_{ch} (fm)
	1^+	2^+	1^-	2^-				
P1I	0.37	0.61	–	-37.9	31	67	3.2	2.41
P2I	0.30	0.55	-1.1	-6.7	44	54	3.0	2.40
P3	0.50		-29.8		30	64	3.6	2.48
P4	0.23		-16.2		67	26	3.3	2.43

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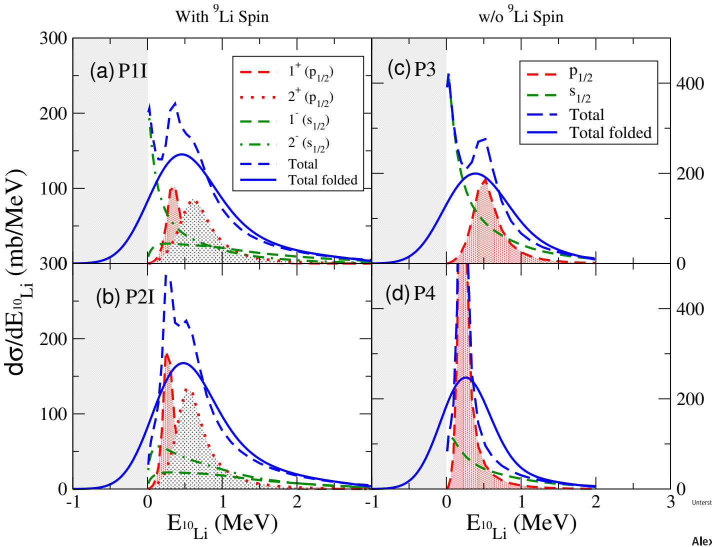
^{10}Li-energy distributions



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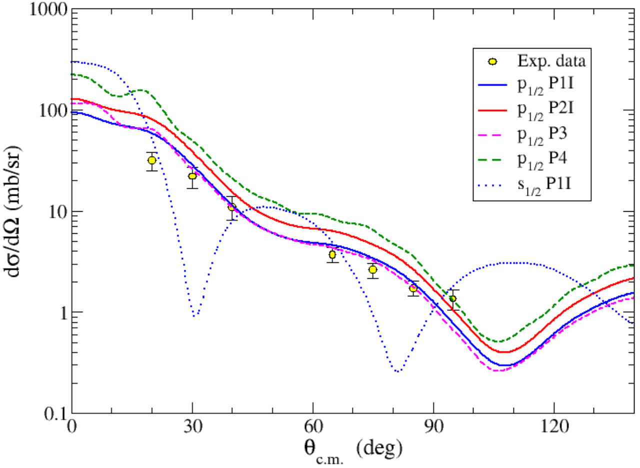
Deuteron angular distribution



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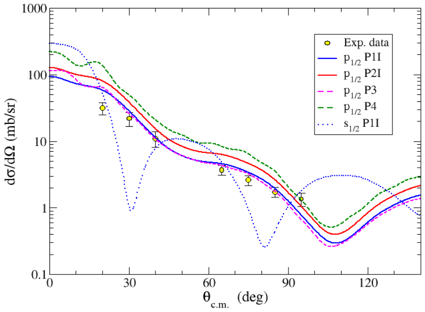
Deuteron angular distribution



- Shape of distribution independent of the model
- Data compatible with p-wave transferred neutron
- Data sensitive to weight of p-wave component
- Mostly insensitive to other characteristics of ^{11}Li wf

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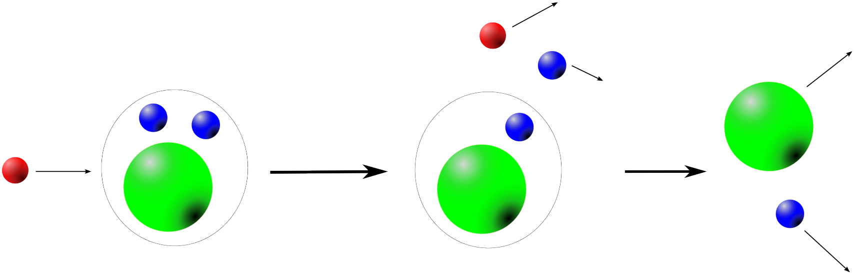
(p, pN) reactions



- A proton and a nucleus collide in such a way that a proton or neutron is removed and the residual nucleus remains.
- High energies ($\sim$ 200-400 MeV) to increase mean free path of nucleon in nucleus.
- It is sometimes referred to as “quasifree” because the main interaction happens between the incoming proton and the extracted nucleon as if it was a free collision.
- This “quasifree” characteristic allows for the use of the participant-spectator model

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Reaction formalism: Transfer to Continuum

- Prior representation of the T-matrix for the process $p+A(B+N+N) \to p+N+C(B+N)$ (participant-spectator model)

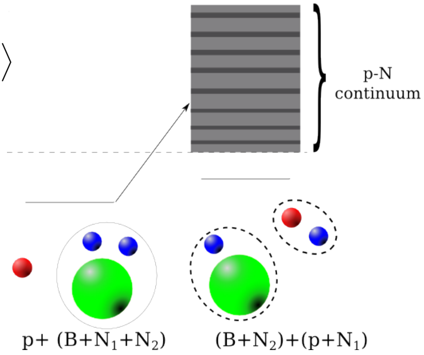
$$\mathcal{T}_{if}^{3b} = \left\langle \Psi_f^{3b(-)} | V_{pN} + U_{pC} - U_{pA} | \varphi_{CA} \chi_{pA}^{(+)} \right\rangle$$

- p-N continuum states discretized in energy bins
  Deuteron included for (p,pn)

$$\phi_n^{j,\pi}(k_n,\vec{r}') = \sqrt{\frac{2}{\pi N}} \int_{k_{n-1}}^{k_n} \phi_n^{j,\pi}(k,\vec{r}')\mathrm{d}k$$

- 3-body final state wavefunction expanded in proton-nucleon states

$$\Psi_f^{3b(-)} \approx \sum_{n,j,\pi} \phi_n^{j,\pi}(k_n,\vec{r}')\chi_{n,j,\pi}^{(-)}(\vec{K_{pn}}',\vec{R}')$$



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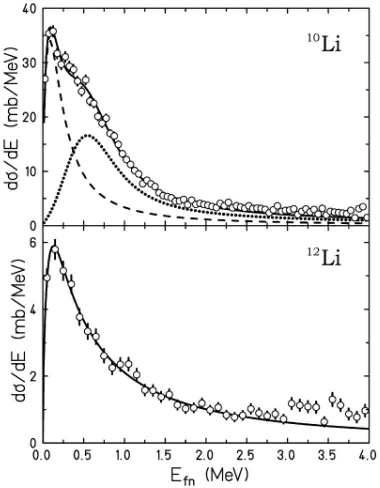
p(^{11}Li,pn)^{10}Li @ 280 MeV/A



Y. Aksyutina *et al*, PLB 666, 430 (2008)

a	$-22.4(4.8)$ fm	E_r	0.566(14) MeV	H target
ϵ	0.352(22) MeV	Γ	0.548(30) MeV	
a	-30^{+12}_{-31} fm	E_r	0.510(44) MeV	C target [5]
ϵ	0.3 MeV	Γ	0.54(16) MeV	
a	$-24 \leqslant a \leqslant -13$ fm	E_r	≈ 0.4 MeV	^{9}Li(d, p) [4]
ϵ	not given	Γ	≈ 0.2 MeV	

- Obtained through fitting
- No prediction of absolute cross section
- Relative weight of s and p weight cannot be known
- Distorsion of distribution due to reaction mechanism?

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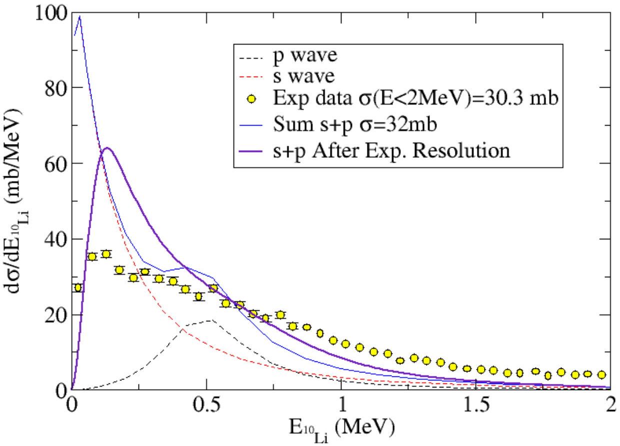
Transfer to the Continuum calculations

- No ^{9}Li Spin model: P3



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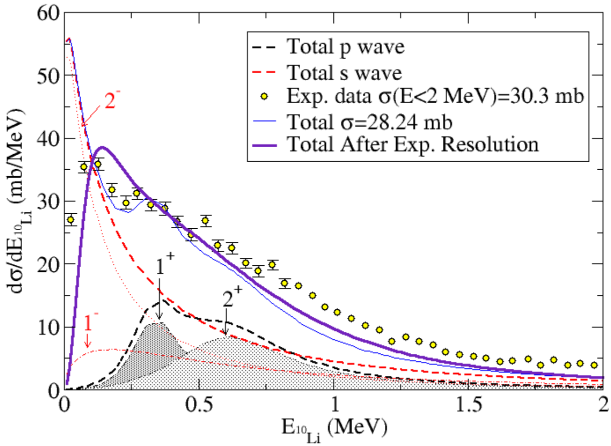
Transfer to the Continuum calculations

- ^{9}Li Spin model: P1I



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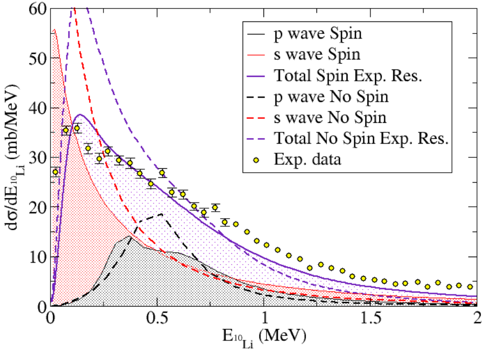
Results of calculation



- The two potentials best agreeing with $p(^{11}\text{Li},d)^{10}\text{Li}$ are presented
- $p(^{11}\text{Li},pn)^{10}\text{Li}$ gives a better agreement with P1I
- Main difference is the splitting of the virtual state
- Experimental resolution smoothes out distribution, hiding features

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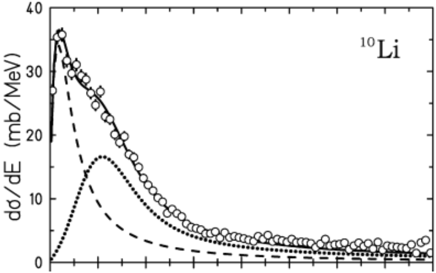
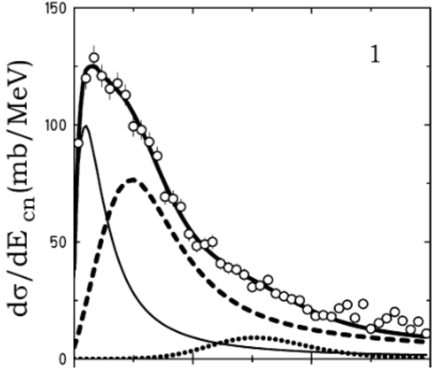
Low-energy d-wave?

$^{11}\mathrm{Li}(p,pn)^{10}\mathrm{Li}$



Y. Aksyutina *et al*, PLB **666**, 430 (2008)

$^{11}\mathrm{Li}(C,X)^{10}\mathrm{Li}$



H. Simon *et al*, NPA **791**, 267 (2007)

- Is there a d-wave resonance at $\sim$1.5 MeV?

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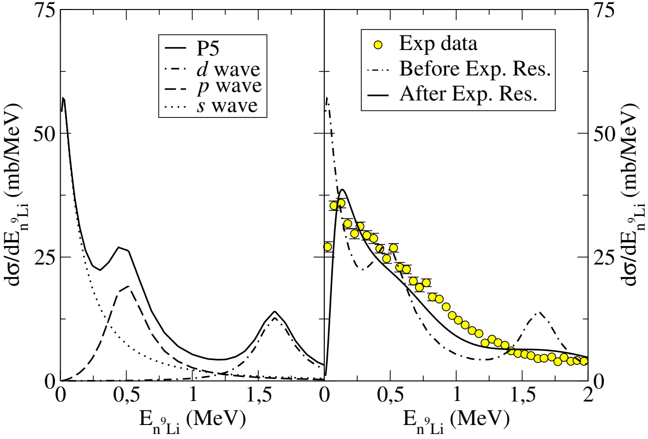
$^{11}\mathrm{Li}(p,pn)^{10}\mathrm{Li}^*$: $I_{^9\mathrm{Li}} = 0 + d$ wave

	E_p (MeV)	a (fm)	E_d (MeV)	$\%p_{1/2}$	$\%s_{1/2}$	$\%d_{5/2}$	r_{mat} (fm)	r_{ch} (fm)
P5	0.50	-29.8	1.5	35	39	23	3.2	2.42



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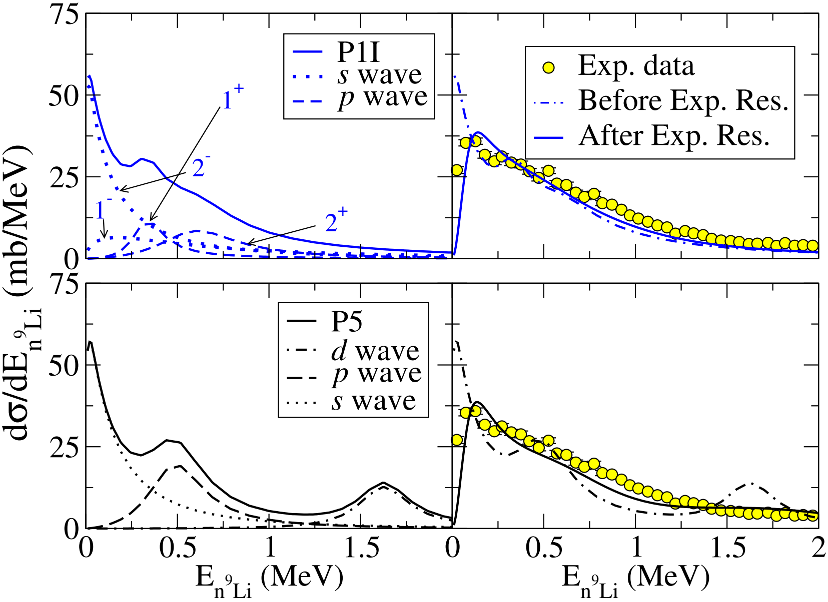
$^{11}\mathrm{Li}(p,pn)^{10}\mathrm{Li}^*$: d-wave?



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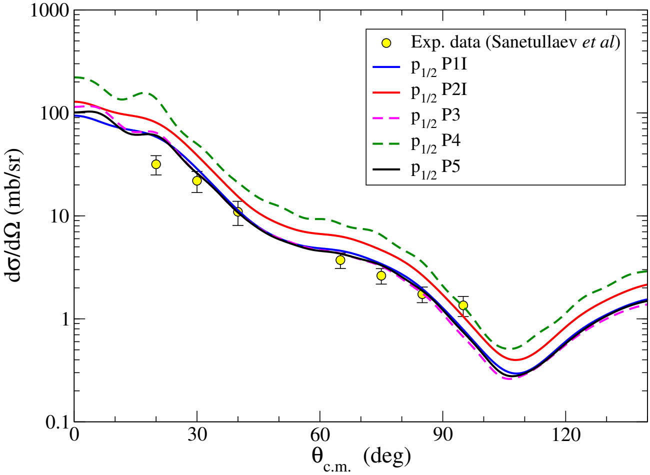
$^{11}\mathrm{Li}(p,d)^{10}\mathrm{Li}^*$: d-wave?



- Are there other reactions to discern between these two models?

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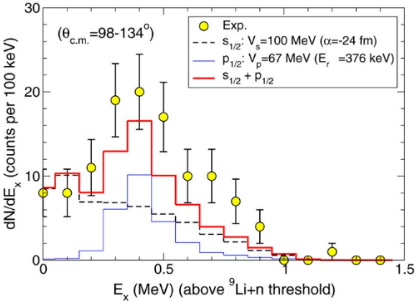
$^9\mathrm{Li}(d,p)^{10}\mathrm{Li}$

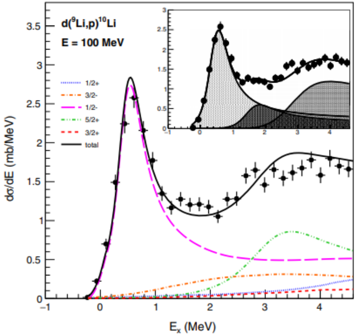
- The explored spectrum is no longer limited by $<^{11}\mathrm{Li(g.s.)}|^{10}\mathrm{Li}>$, instead $<^{9}\mathrm{Li(g.s.)}|^{10}\mathrm{Li}>$

$d(^9\mathrm{Li},p)^{10}\mathrm{Li}$ at 2.4 MeV/A



H.B. Jeppesen *et al*, PLB **642**, 449 (2006)

$d(^9\mathrm{Li},p)^{10}\mathrm{Li}$ at 11 MeV/A



M. Cavallaro *et al*, PRL **118**, 012701 (2017)

- Is there a virtual state in ^{10}Li?
- Why do the spectra look so different?

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Transfer to Continuum, slightly different

- Prior representation of the T-matrix for the process $d + B \to p + A(N+B)$ (participant-spectator model)

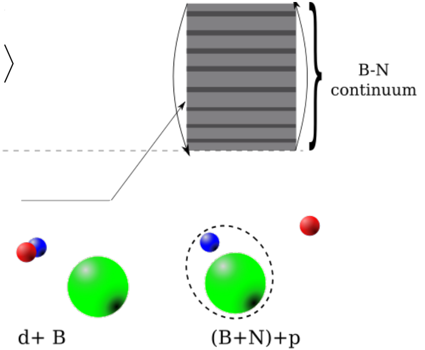
$$\mathcal{T}_{if}^{3b} = \left\langle \Psi_f^{3b(-)} | V_{NB} + U_{pB} - U_{dB} | \varphi_{AB} \chi_{dB}^{(+)} \right\rangle$$

- N-B continuum states discretized in energy bins and allowed to couple

$$\phi_n^{j,\pi}(k_n, \vec{r}') = \sqrt{\frac{2}{\pi N}} \int_{k_{n-1}}^{k_n} \phi_n^{j,\pi}(k, \vec{r}') \mathrm{d}k$$

- 3-body final state wavefunction expanded in neutron-^{9}Li states

$$\Psi_f^{3b(-)} \approx \sum_{n,j,\pi} \phi_n^{j,\pi}(k_n, \vec{r}') \chi_{n,j,\pi}^{(-)}(\vec{K_{NB}}', \vec{R}')$$



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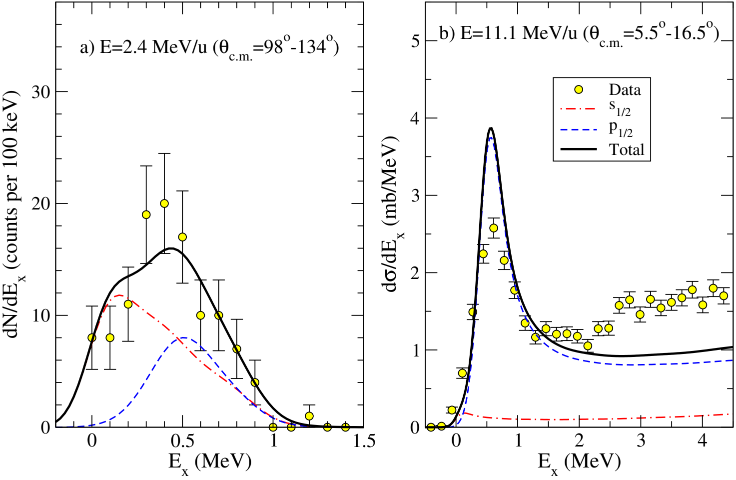
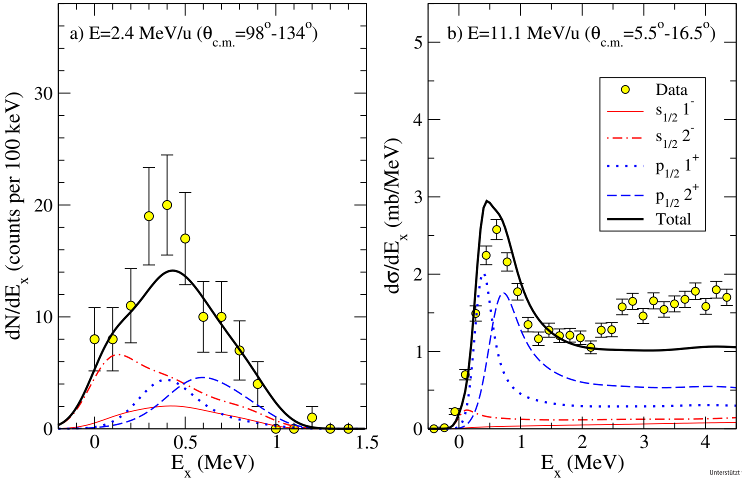
P3: warmup



- At 2.4 MeV/A, the maximum energy is too low to explore the spectrum over ∼1 MeV, we will not get the d-wave resonance from there

P1I



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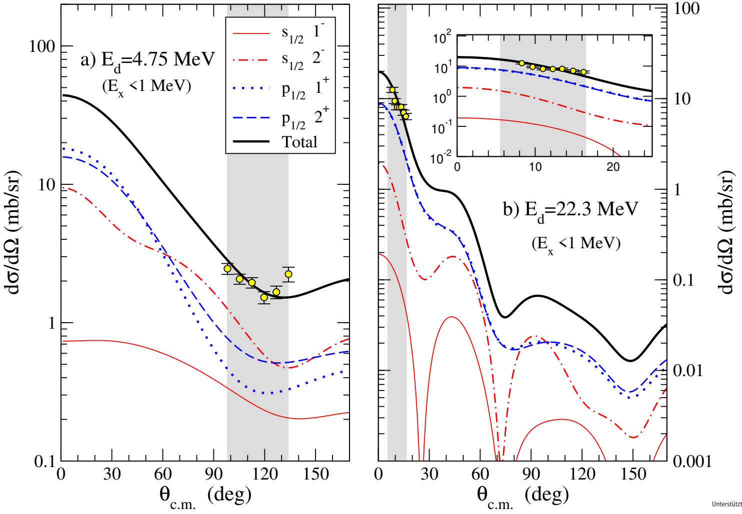
Different spectra: Angular ranges



- Different angular ranges emphasize different components

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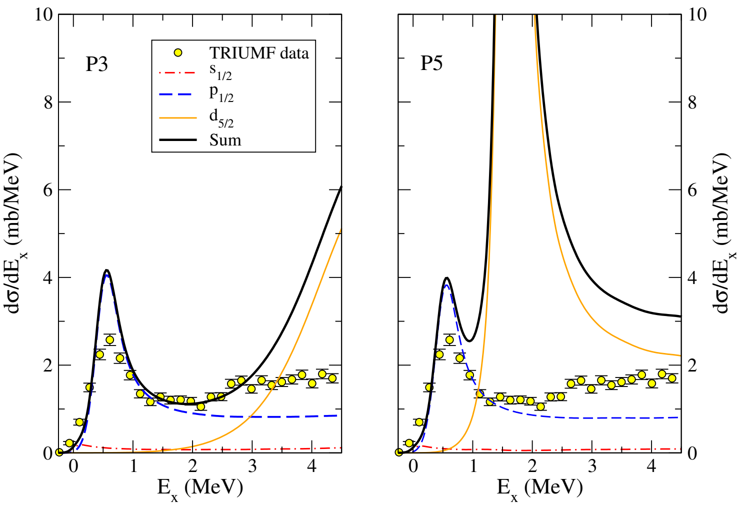
P3 vs P5: Low-energy d-wave resonance?



- Strong coupling to d-wave states, this rules out a defined low-energy resonance (it could be fragmented)

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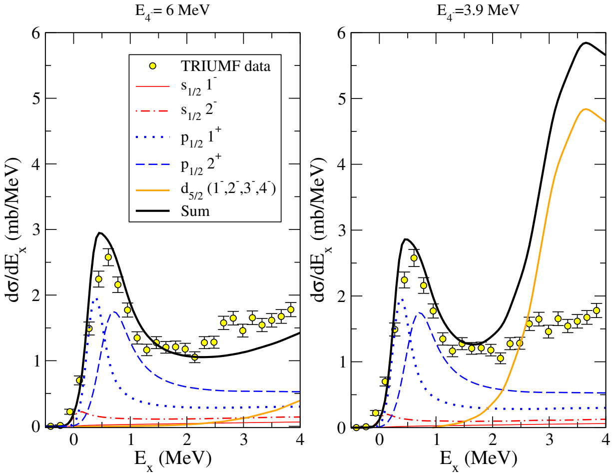
P1I: Ambiguities in the $l = 2$ contribution



- $(1^-, 2^-, 3^-, 4^-)$ multiplet, too many free parameters to adjust phenomenologically, and too few data.

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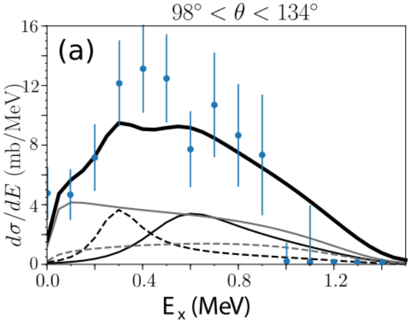
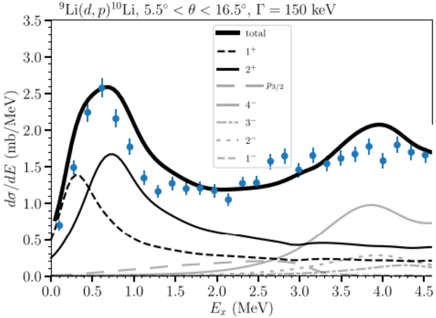
NFT calculations

$d(^{9}\text{Li},p)^{10}$Li at 2.4 MeV/A



$d(^{9}\text{Li},p)^{10}$Li at 11 MeV/A



F. Barranco *et al*, PRC **101**, 031305(R) (2020)

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Conclusions

- A method to obtain $< C + N|C + N + N >$ overlaps for Borromean nuclei from 3-body structure calculations has been developed and applied to ^{11}Li.
- These overlaps can be used for various reactions, as long as they verify the participant-spectator model for the components of the Borromean nucleus
- The method has been applied for the reactions $p(^{11}\mathrm{Li}, d)^{10}\mathrm{Li}$ at 5.7 MeV/A, $p(^{11}\mathrm{Li},pn)^{10}\mathrm{Li}$ at 280 MeV/A, and $^{9}\mathrm{Li}(d, p)^{10}\mathrm{Li}$ at 2.4 and 11.1 MeV/A obtaining remarkable agreement for all reactions, using the same n-^{9}Li interaction.
- The (p, d) transfer reaction is found to be sensitive to the angular momentum and spectroscopic factor of the transferred nucleon mostly.
- The (p, pn) nucleon removal reaction seems to favour a description including the spin of ^{9}Li and splitting of the virtual state and resonance or the inclusion of low-energy d-wave resonance
- The $^{9}\mathrm{Li}(d, p)^{10}\mathrm{Li}$ transfer rejects low-energy d-wave resonance, but the d-wave spectrum is still not well understood (^{10}Li always laughs last).

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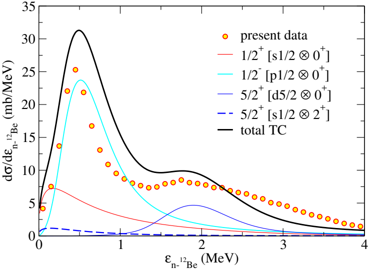
^{14}Be$(p,pn)^{13}$Be*, A. Corsi *et al*, PLB **797**, 134843 (2019)



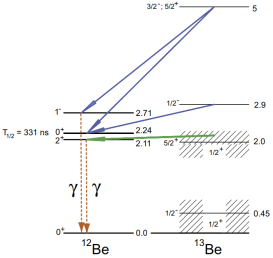
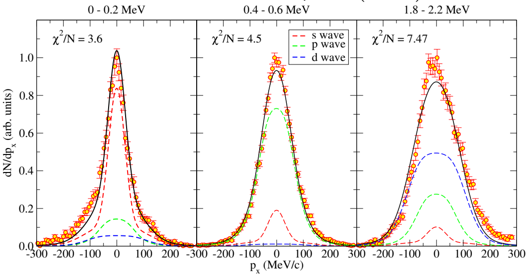


Fig. from Y. Aksyutina *et al*, *PRC* **87**, *064316* (2013)



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