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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Ab initio Leading-Order Effective Interactions for Elastic Scattering of Nucleons from Light Nuclei

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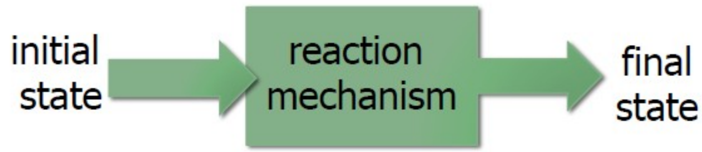
Exotic Nuclei are usually short lived:

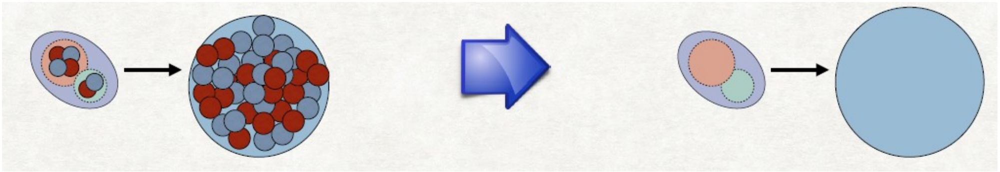
Have to be studied with reactions in inverse kinematics

e.g. direct reaction:



Challenge:

- **In the continuum, theory can solve the few-body problem exactly.**



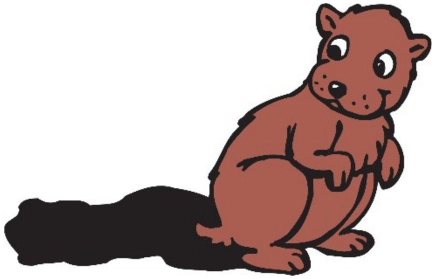
Many-body problem

Few-body problem

Example (d,p) Reactions:

Reduce Many-Body to Few-Body Problem

Solve few-body problem



Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

Challenges & Opportunities

- **Nucleon-nucleon interaction believed to be well known:**
  today: chiral interactions

- **Effective proton (neutron) nucleus interactions:**
  - purely phenomenological optical potentials fitted to data
  - optical potentials with theoretical guidance
  - **microscopic optical potentials**

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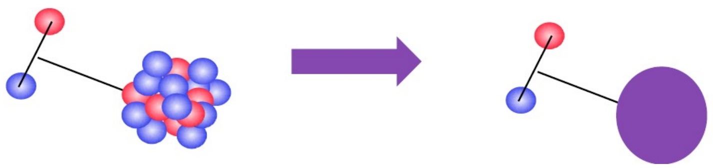
Isolate relevant degrees of freedom

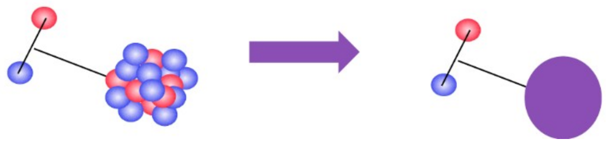


Formally: separate Hilbert space into **P** and **Q** space, and calculate in **P** space

Projection on **P** space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly
(Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Isolate relevant degrees of freedom



Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly
(Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Often used:

Phenomenological optical potentials

Either fitted to a large global data set OR to a restricted data set

Most general form of optical potential

- $\sum_i [V_{A,Z,N,E}(r) + i\, W_{A,Z,N,E}(r)]\ \mathrm{Operator}_{(i)}$
- **Functions are of Woods-Saxon type**

Have central and spin orbit term

Fit cross sections, angular distributions polarizations, for a set of nuclei (lightest usually ^{12}C).

No connection to microscopic theory

Dispersive optical models have some connection to structure

Today: huge progress in *ab initio* structure calculations

➡ Goal: effective interaction from *ab initio* methods

Start from many-body Hamiltonian with 2 and 3 body forces

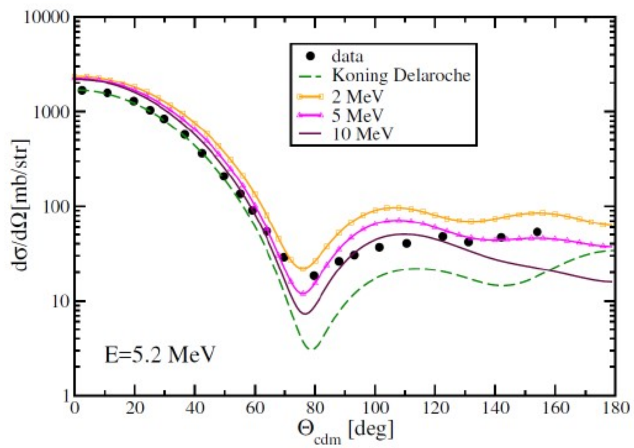
Theoretical foundations laid by Feshbach and Watson in the 1950s

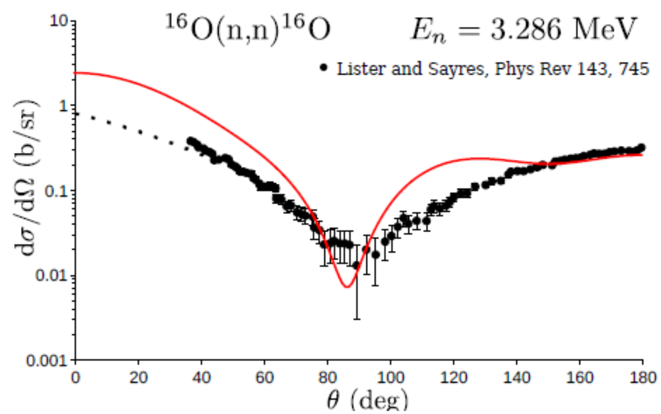
Feshbach:

► effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

$^{40}Ca(n,n)^{40}Ca$

energy ~ 10 MeV





Rotureau, Danielewicz, Hagen, Jansen, Nunes
arXiv: 1808.04535 and PRC 95, 024315 (2017)

Idini, Barbieri, Navratil
J.Phys.Conf. 981. 012005 (2018)
Acta Phys. Polon. B48, 273 (2017)

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Today's Goal: effective interaction from *ab initio* methods

Start from many-body Hamiltonian with 2 (and 3) body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

 effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

energy ~ 10 MeV

Watson:

► **Multiple scattering expansion, e.g. spectator expansion (current truncation to 2 active particles)**

Spectator Expansion:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:

- **particles active in the reaction**
- **antisymmetrized in active particles**

"fast reaction", i.e. ≥ 100 MeV

Elastic Scattering (Watson approach)

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P} = |\Phi_0\rangle\langle\Phi_0|$
  - With $\mathbf{1=P+Q}$ and $\mathbf{[P,G_0]=0}$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$

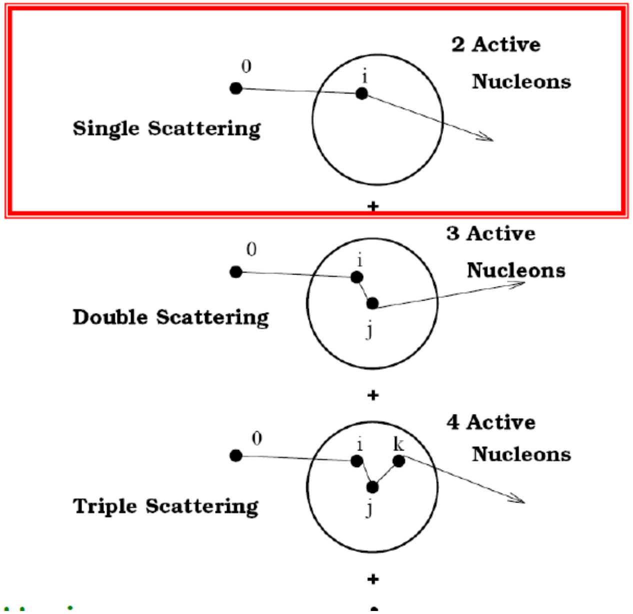
$$\mathbf{T = U + U\,G_0(E)\,P\,T}$$

$$\mathbf{U = V + V\,G_0(E)\,Q\,U}$$

⇐ **effective (optical) potential**

Exact expression

Spectator Expansion of U :



1st order: single scattering: $\mathbf{U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}}$

Chinn, Elster, Thaler, PRC 47, 2242 (1993)

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$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- Deal with **Q** **(this prevents to use free NN t-matrix here)**
  - Define "two-body" operator t_{0i}^{free} by
  - $t_{0i}^{free} = v_{0i} + v_{0i}\, G_0(e)\, t_{0i}^{free}$
  - and relate via integral equation to τ_{oi}
  - $\tau_{oi} = t_{0i}^{free} - t_{0i}^{free}\, G_0(e)\, \tau_{oi}$ **[integral equation]**
  - keeps iso-spin character of optical potential
  - $\hat{U}^{(1)} = \Sigma^{A}_{i=1} \tau_{oi} =: N\, \tau_n + Z\, \tau_p$

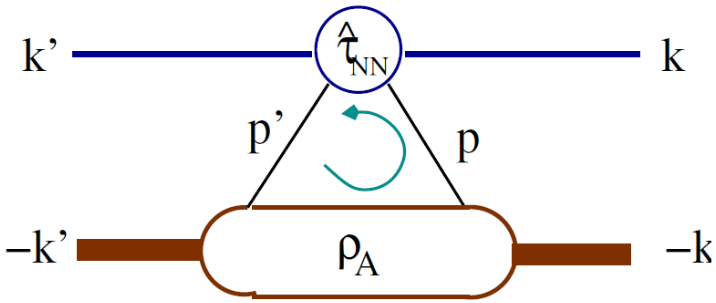
Integral ≡ average

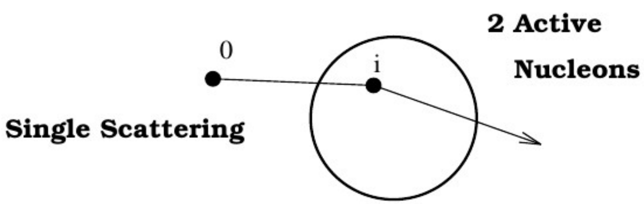
Neutron and proton contributions are cleanly separated
Important for N≠Z nuclei

$$t_{pp} \neq t_{np} \quad \text{and } \rho_p \neq \rho_n$$

$G_0(E)$ ≡ many-body operator → use e.g. closure to obtain two-body propagator

Computing the first order folding potential $\hat{U}^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN scattering amplitudes

Nuclear one-body density

$$\widehat{U}(q,\mathcal{K}_{NA},\epsilon) = \sum_{\alpha=n,p}\sum_{K_s}\int d^3\mathcal{K}\;\eta(q,\mathcal{K},\mathcal{K}_{NA})\;\hat{\tau}_{\alpha}^{K_s}\left(q,\frac{1}{2}\left(\frac{A+1}{A}\mathcal{K}_{NA}-\mathcal{K}\right);\varepsilon\right)\;\rho_{\alpha}^{K_s}\left(\mathcal{K}-\frac{A-1}{A}\frac{q}{2},\mathcal{K}+\frac{A-1}{A}\frac{q}{2}\right)$$

$$q = p' - p$$
$$\mathcal{K} = \frac{1}{2}(p' + p)$$

Same NN Interaction can now be used for NN t-matrix and the one-body density matrix (***ab initio***)

Effective Potential is non-local and energy dependent

Details of implementation designed for energies ≥ 100 MeV

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NN amplitude: $f_{NN}(k'k;E) \approx \langle k' | t_{NN}(\varepsilon) | k \rangle$

with $q = k' - k$

$K_{NN} = \frac{1}{2}(k' + k)$

Most general form

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

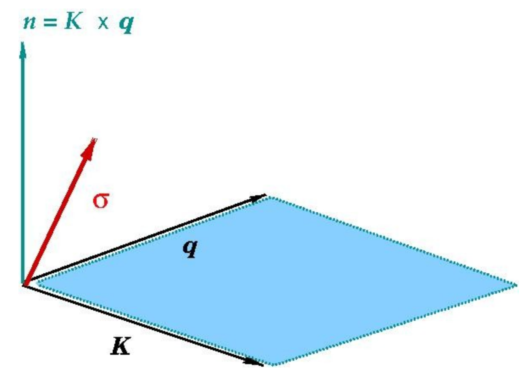
$$\overline{\mathrm{M}}(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) = A(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, \mathbf{1} + i \cdot C(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, (\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN})$$

- -

$$+ D(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \right]$$ Off-shell

Evaluating scalar products with $\boldsymbol{\sigma}^{(i)}$

requires **spin-dependent one-body density matrices**



Spin-dependent nonlocal one-body density matrix

define

$$\rho_{q_s}^{K_s}(p,p') = \left\langle \Phi' \left| \sum_{i=1}^{A} \delta^3(p_i - p)\delta^3(p'_i - p')\sigma_{q_s}^{(i)K_s} \right| \Phi \right\rangle$$

$$K_s = 0 \; : \quad (\hat{\sigma})_0^0 = 1$$ ← Scalar OBDM

$$K_s = 1 \; : \quad (\hat{\sigma})_0^1 = \sigma_z$$ ← Spin-dependent OBDM

$$: \quad (\hat{\sigma})_{-1}^1 = \frac{1}{\sqrt{2}}(\sigma_x - i\sigma_y)$$

$$: \quad (\hat{\sigma})_1^1 = -\frac{1}{\sqrt{2}}(\sigma_x + i\sigma_y)$$

Needed: Spin-projected momentum distribution [here ~ $(\sigma^{(i)} \cdot \hat{n})$]

$$S_n(p,p') = \sum_{q_s} \left\langle \Phi \left| \sum_{i=1}^{A} \delta^3(p_i - p)\delta^3(p'_i - p')\hat{\sigma}_{q_s}^{(i)K_s=1} \right| \Phi \right\rangle (-1)^{q_s} (\hat{n}^1_{t.i.})_{-q_s}.$$

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Spin-projected momentum distribution ~ $(\boldsymbol{\sigma}^{(i)} \cdot \hat{\boldsymbol{n}})$

Evaluation based on NCSM matrix elements
Change of variables to $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$ $\boldsymbol{\mathcal{K}} = \frac{1}{2}(\boldsymbol{p}' + \boldsymbol{p})$ to remove CoM

$$S_n(\boldsymbol{q},\boldsymbol{\mathcal{K}}) = \sum_{q_s}(-1)^{-q_s}\sqrt{\frac{4\pi}{3}}Y^1_{-q_s}(\hat{\boldsymbol{n}}) \sum_{nljn'l'j'} \sum_{K_l=|l-l'|}^{l+l'} \sum_{k_l=-K_l}^{K_l} \sum_{Kk} \langle K_l k_l 1 q_s | K k\rangle$$
$$(-1)^{J-M} \begin{pmatrix} J & K & J \\ -M & k & M \end{pmatrix} (-1)^{-l}\hat{j}\hat{j}'\hat{s}\hat{1}\hat{K}_l \begin{Bmatrix} l' & l & K_l \\ s & s & 1 \\ j' & j & K \end{Bmatrix} (-i)^{l+l'}$$
$$\sum_{n_q,n_\mathcal{K},l_q,l_\mathcal{K}} \langle n_\mathcal{K} l_\mathcal{K}, n_q l_q : K_l | n'l', nl : K_l\rangle_{d=1} R_{n_\mathcal{K} l_\mathcal{K}}(\mathcal{K}) R_{n_q l_q}(q) \mathcal{Y}^{*l_q l_\mathcal{K}}_{K_l k_l}(\hat{q},\hat{\mathcal{K}})$$
$$\underline{\left\langle A\lambda J \left|\left| (a^\dagger_{n'l'j'}\tilde{a}_{nlj})^{(K)} \right|\right| A\lambda J \right\rangle} e^{\frac{1}{4A}b^2 q^2},$$

reduced NCSM matrix elements

technical details in Burrows et al. arXiv:2005.00111

Spin-projected momentum distribution ~ $(\sigma^{(i)} \cdot \hat{n})$

Evaluation based on NCSM matrix elements

Change of variables to $q = p' - p \quad \mathcal{K} = \frac{1}{2}(p' + p)$ nove CoM

$$S_n(q,\mathcal{K}) = \sum_{q_s} (-1)^{-q_s} \sqrt{\frac{4\pi}{3}} Y^1_{-q_s}(\hat{n}) \sum_{nljn'l'j'} \sum_{K_l=|l-l'|}^{l+l'} \sum_{k_l=-K_l}^{K_l} \sum_{Kk} \langle K_l k_l 1 q_s | K k \rangle$$
$$(-1)^{J-M} \begin{pmatrix} J & K & J \\ -M & k & M \end{pmatrix} (-1)^{-l} \hat{j}\hat{j}'\hat{s}\hat{1}\hat{K}_l \begin{Bmatrix} l' & l & K_l \\ s & s & 1 \\ j' & j & K \end{Bmatrix} (-i)^{l+l'}$$
$$\sum_{n_q,n_\mathcal{K},l_q,l_\mathcal{K}} \langle n_\mathcal{K} l_\mathcal{K}, n_q l_q : K_l | n'l', nl : K_l \rangle_{d=1} R_{n_\mathcal{K} l_\mathcal{K}}(\mathcal{K}) R_{n_q l_q}(q) \mathcal{Y}^{*l_q l_\mathcal{K}}_{K_l k_l}(\hat{q}, \hat{\mathcal{K}})$$
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reduced NCSM matrix elements

technical details in Burrows et al. arXiv:2005.00111

Similarly: spin-projected momentum distributions $(\sigma^{(i)} \cdot \hat{q})$ and $(\sigma^{(i)} \cdot \hat{\mathcal{K}})$

Both give zero contribution due to parity arguments

NN amplitude: $f_{NN}(k'k;E) \approx \langle k' | t_{NN}(\varepsilon) | k \rangle$

with $q = k' - k$

$K_{NN} = \frac{1}{2}(k' + k)$

Most general form

NN amplitudes contributing to effective nucleon-nucleus interaction:

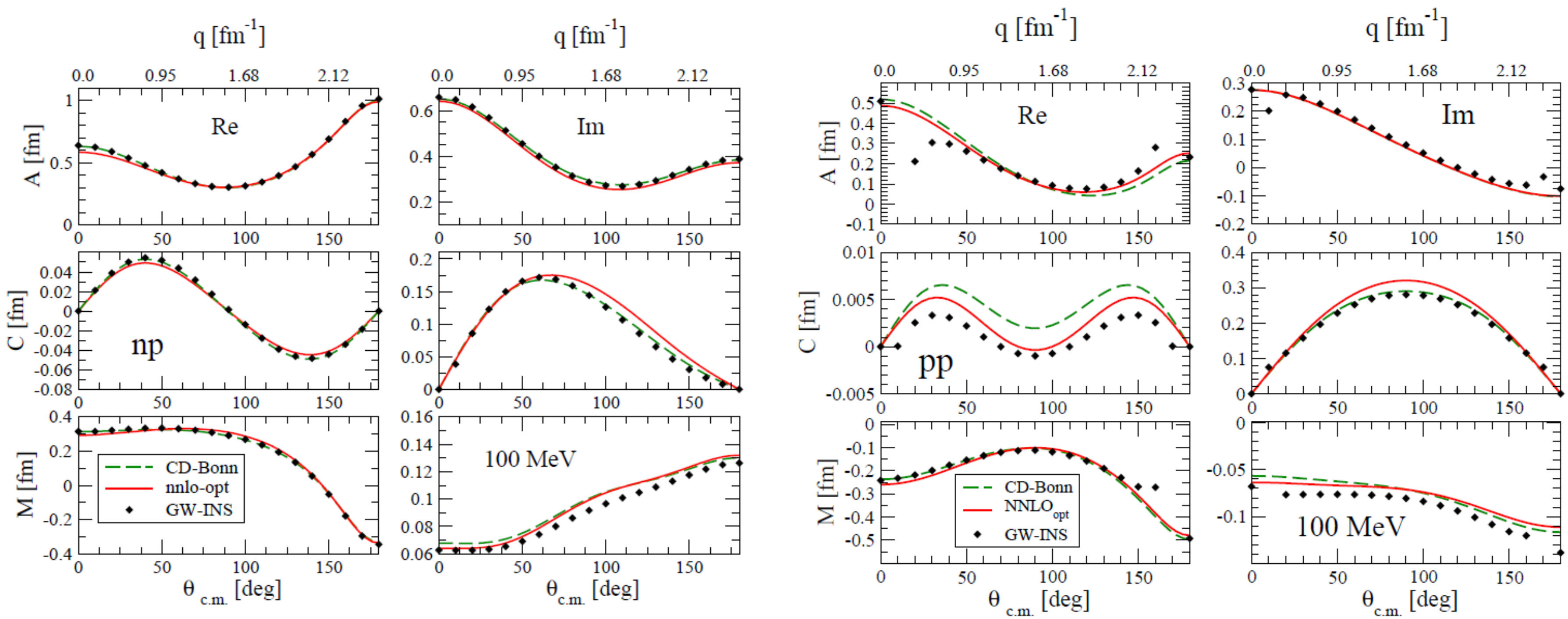
Approximation neglecting spin of struck nucleon

$$
\begin{aligned}
\overline{M}(q,\mathcal{K}_{NN},\epsilon) = {} & A(q,\mathcal{K}_{NN},\epsilon)\mathbf{1}\otimes\mathbf{1} \\
& + iC(q,\mathcal{K}_{NN},\epsilon)\left(\boldsymbol{\sigma}^{(0)}\cdot\hat{n}\right)\otimes\mathbf{1} \\
& + iC(q,\mathcal{K}_{NN},\epsilon)\,\mathbf{1}\otimes\left(\boldsymbol{\sigma}^{(i)}\cdot\hat{n}\right) \\
& + M(q,\mathcal{K}_{NN},\epsilon)(\boldsymbol{\sigma}^{(0)}\cdot\hat{n})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{n}) \\
& + [G(q,\mathcal{K}_{NN},\epsilon) - H(q,\mathcal{K}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{q})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{q}) \\
& + [G(q,\mathcal{K}_{NN},\epsilon) + H(q,\mathcal{K}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathcal{K}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathcal{K}}) \\
& + D(q,\mathcal{K}_{NN},\epsilon)\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{q})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathcal{K}}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathcal{K}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{q})\right]
\end{aligned}
$$

Wolfenstein Amplitudes A, C, M

$NNLO_{opt}$
fitted to
E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}



$NNLO_{opt}$ A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G. R. Jansen, R. Machleidt, W. Nazarewicz, *et al.*, Phys. Rev. Lett. **110**, 192502 (2013).

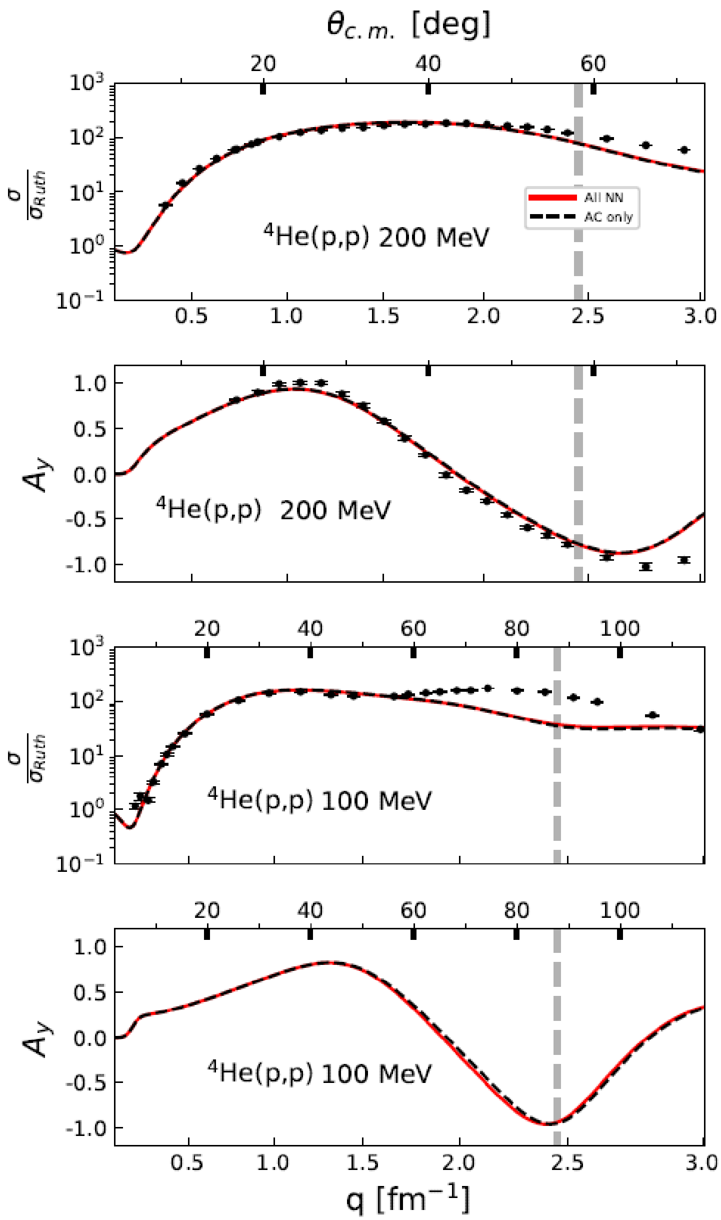
CD-Bonn R. Machleidt, Phys. Rev. **C63**, 024001 (2001)

GW-INS R. L. Workman, W. J. Briscoe, and I. I. Strakovsky, Phys. Rev. **C94**, 065203 (2016).

^{4}He $N_{max}=18$

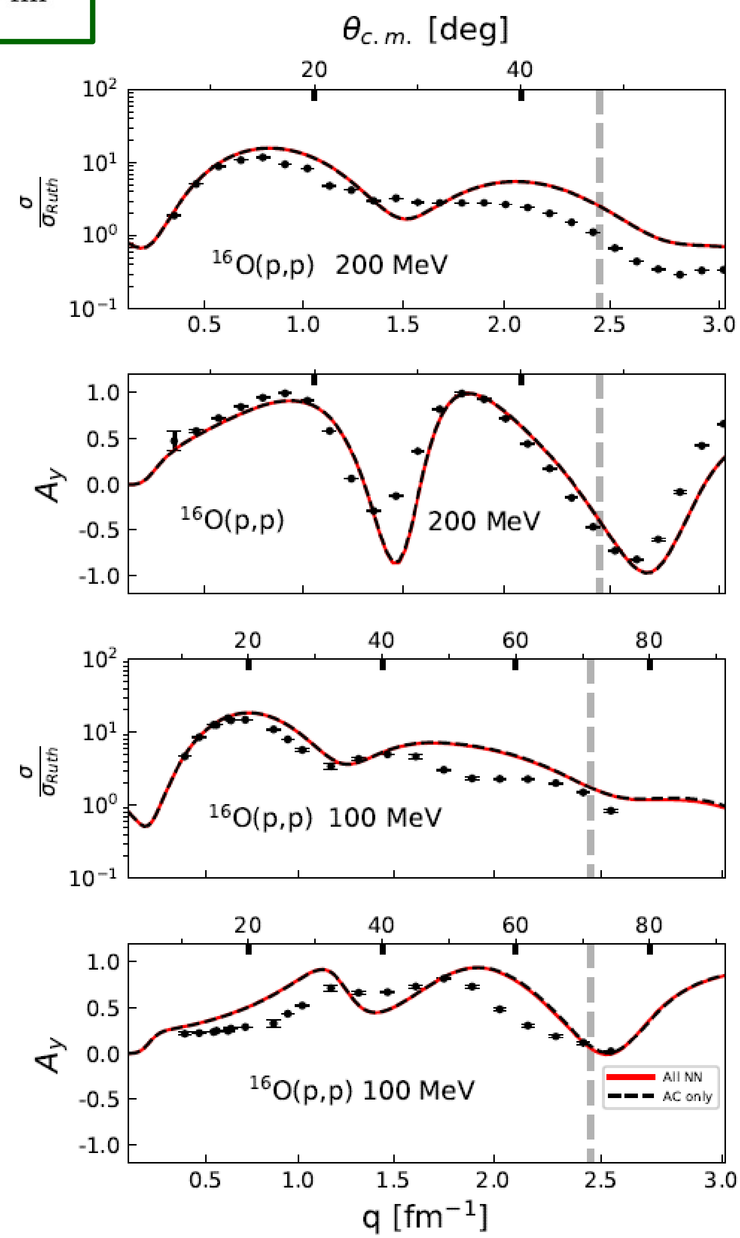
$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$



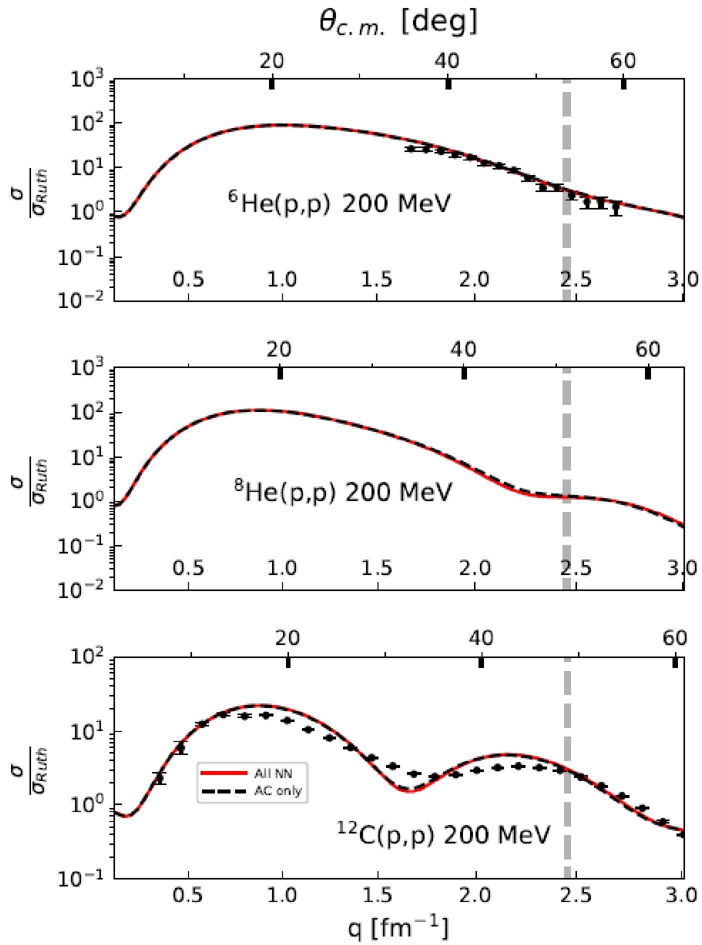
closed-shell nuclei

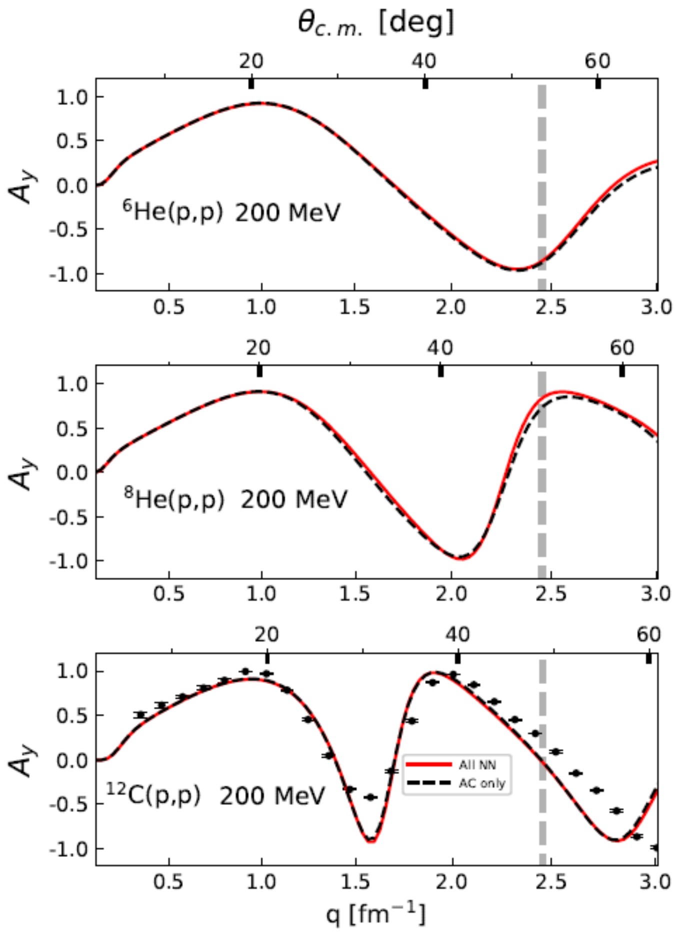
$N_{max}=10$ ^{16}O



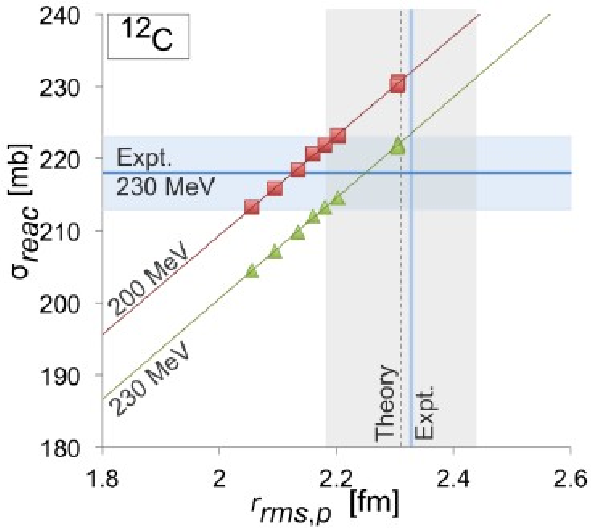
$\hbar\omega=20$

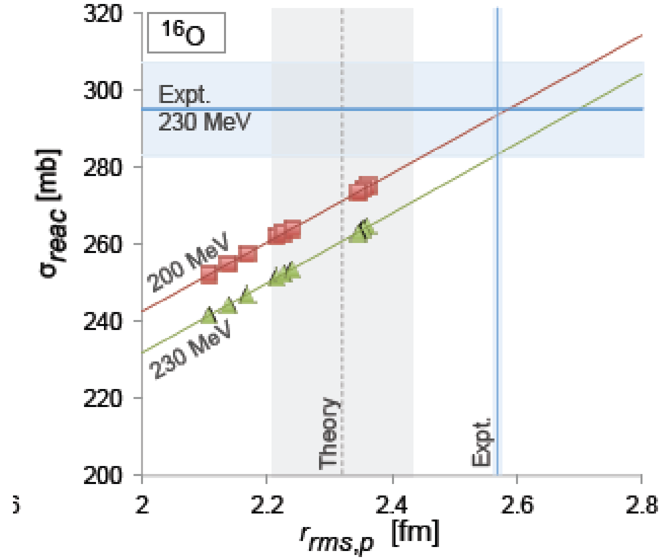
Open-shell nuclei at 200 MeV





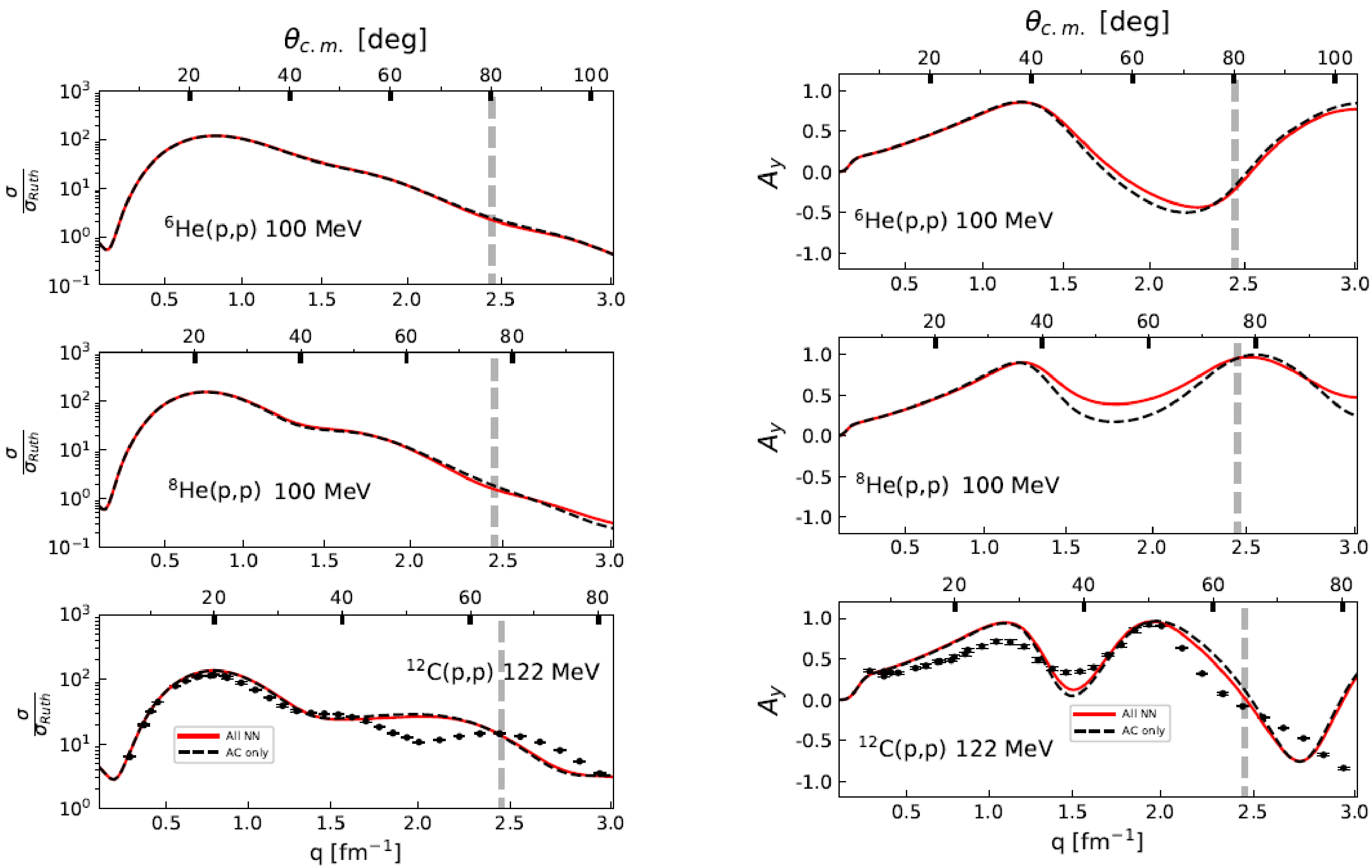
Reaction cross section and extracted point proton radius



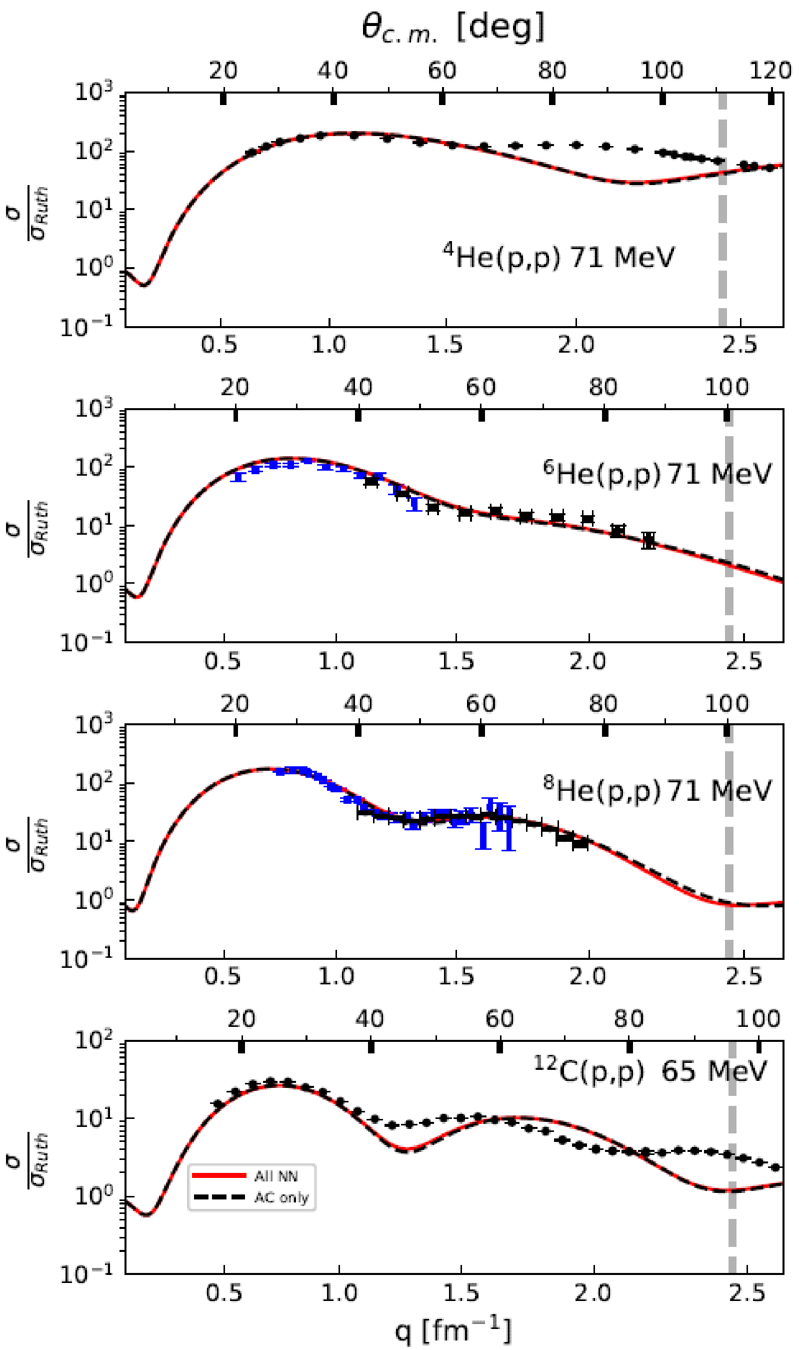


calculations are performed for $N_{\max} = 6,\ 8$, and 10, and for $\hbar\omega = 16,\ 20$, and 24 MeV.

Open-shell nuclei at 100 MeV



Energies lower than 100 MeV



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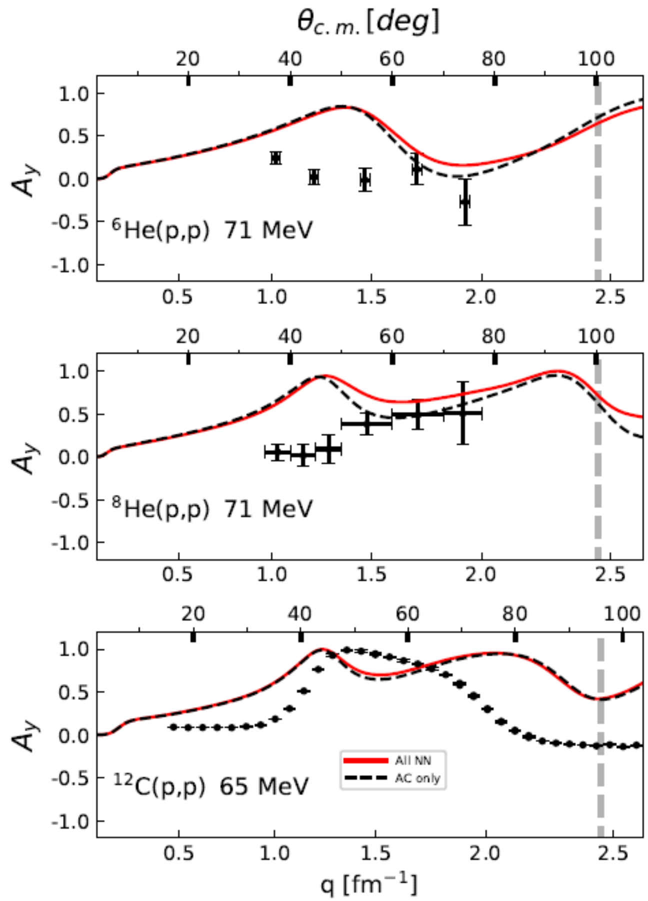
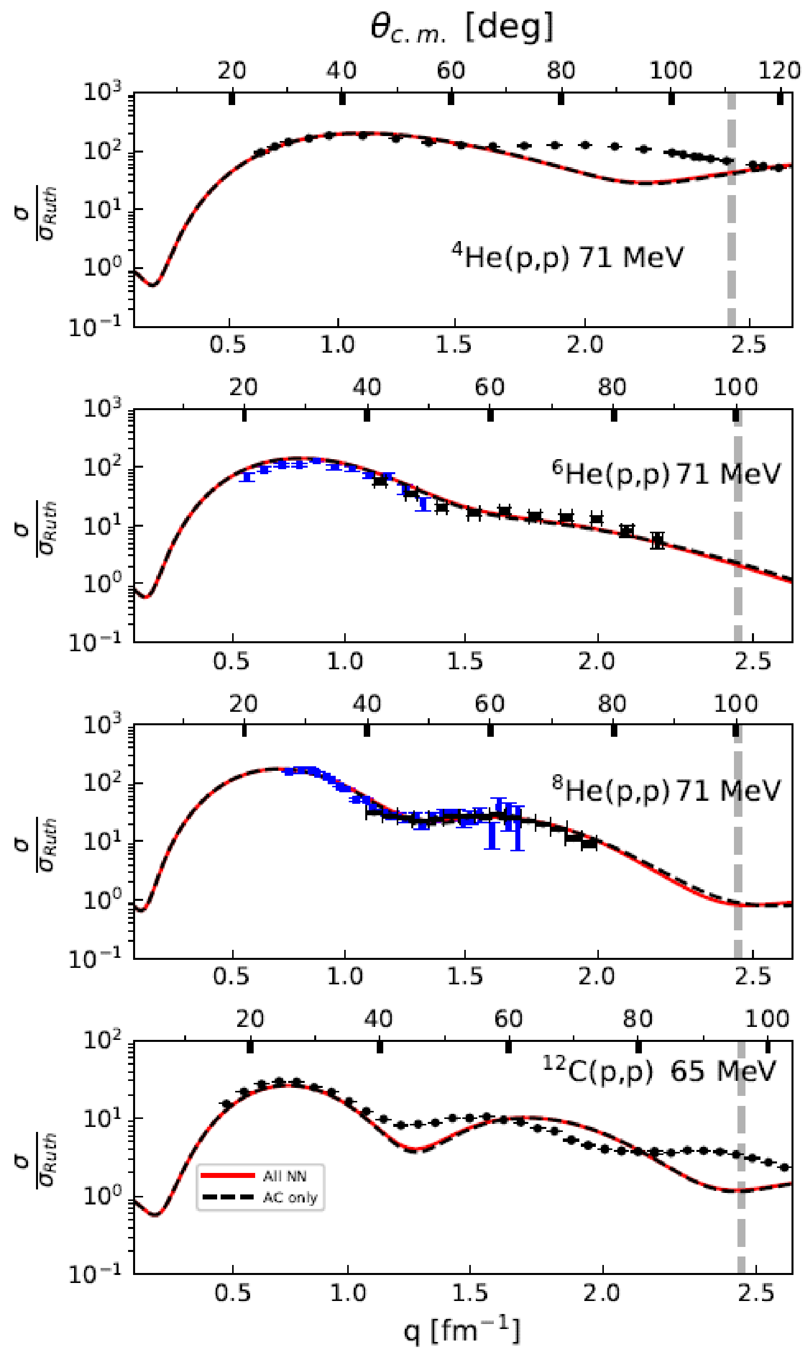
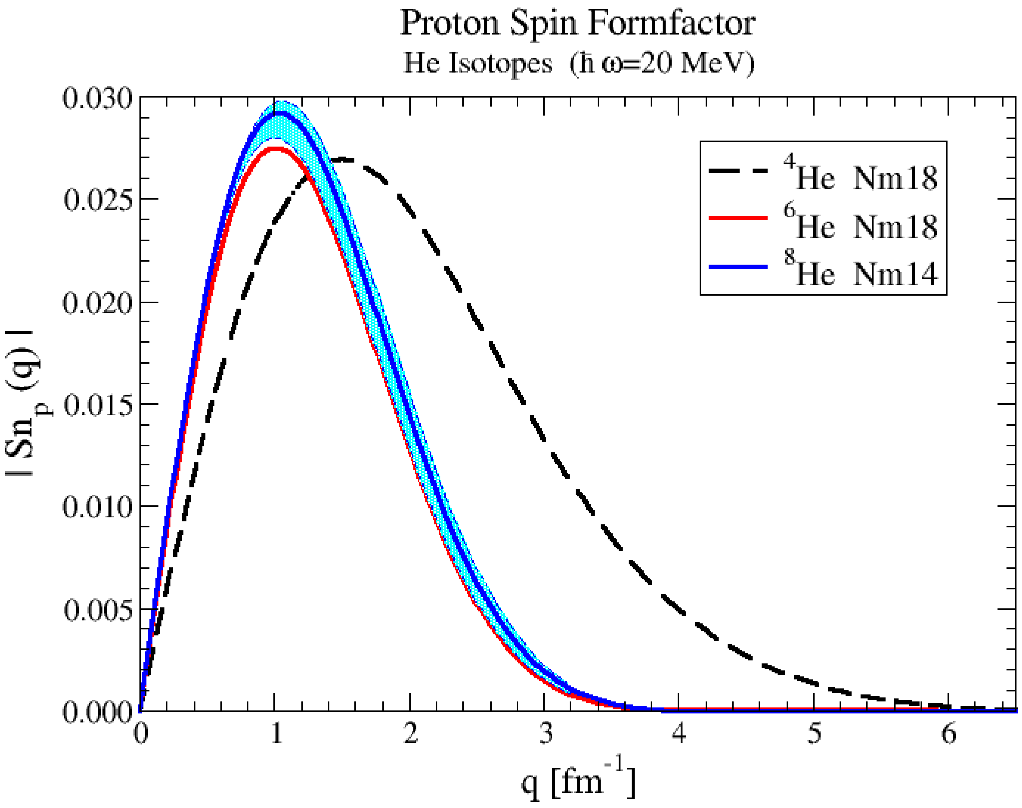
Energies lower than 100 MeV



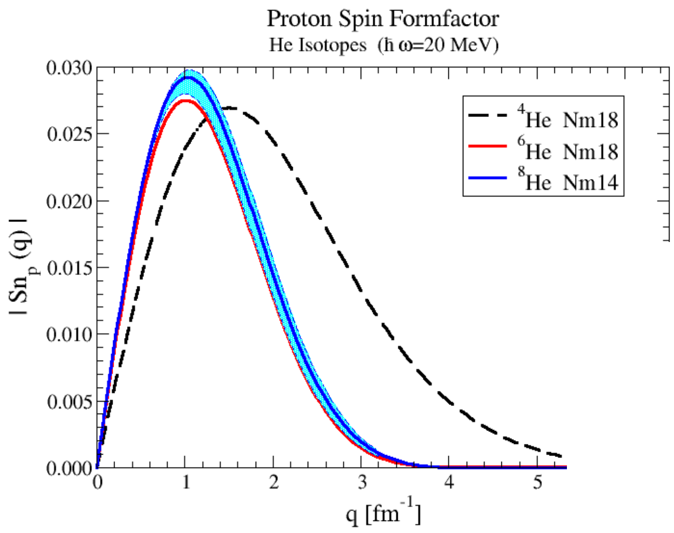


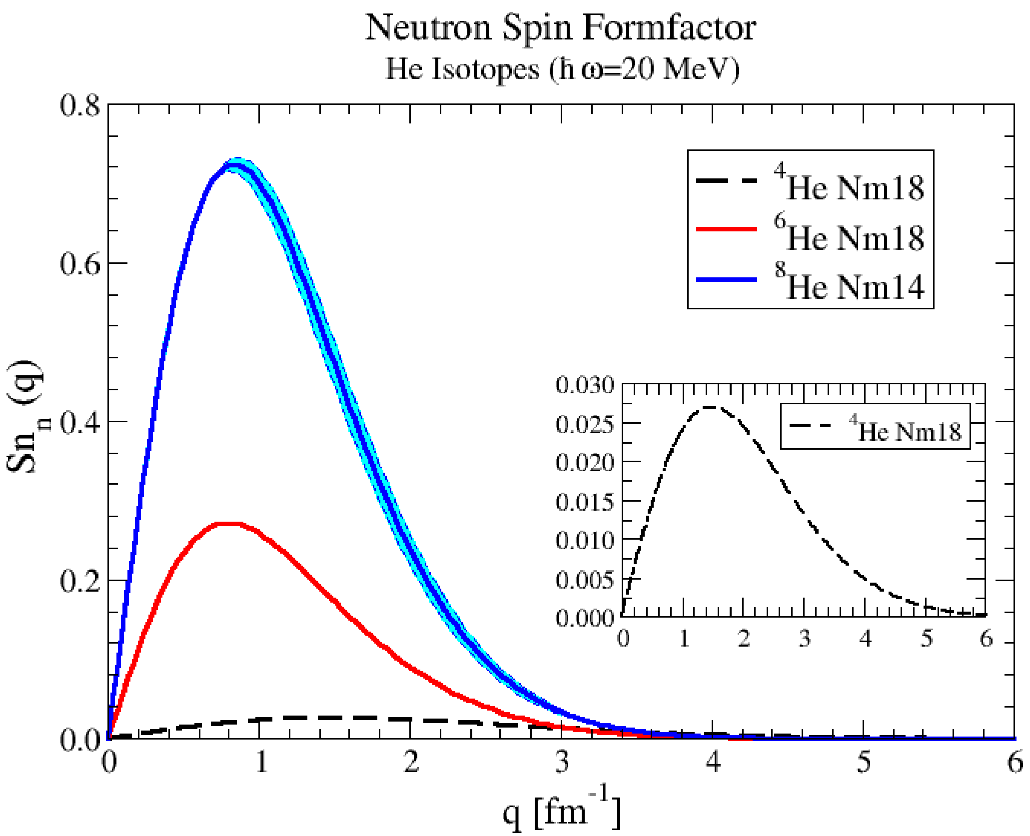
Can we learn more from the spin projected momentum distribution?

Define: $$S_n(\mathbf{q}) = \int d^3K S_n(\mathbf{q},\mathbf{K})$$ == spin form factor

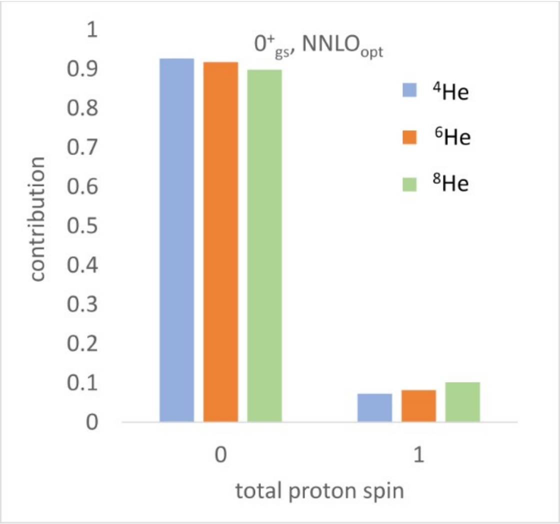


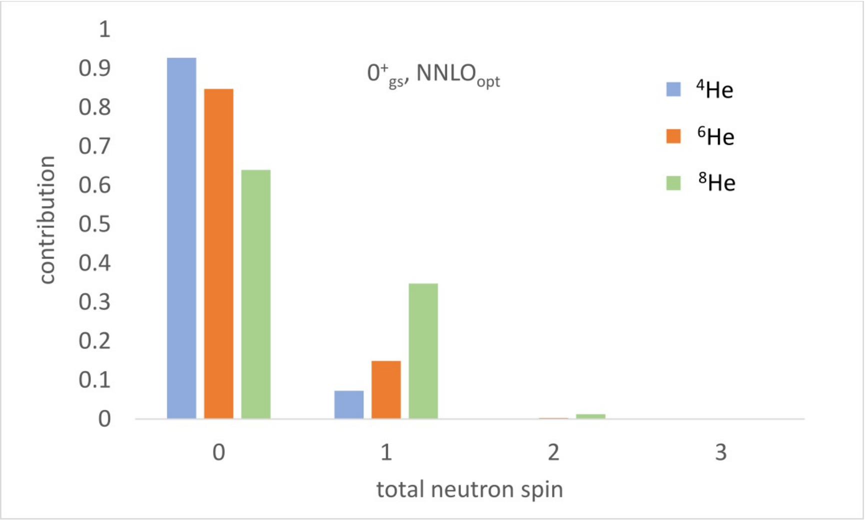
Spin form factors

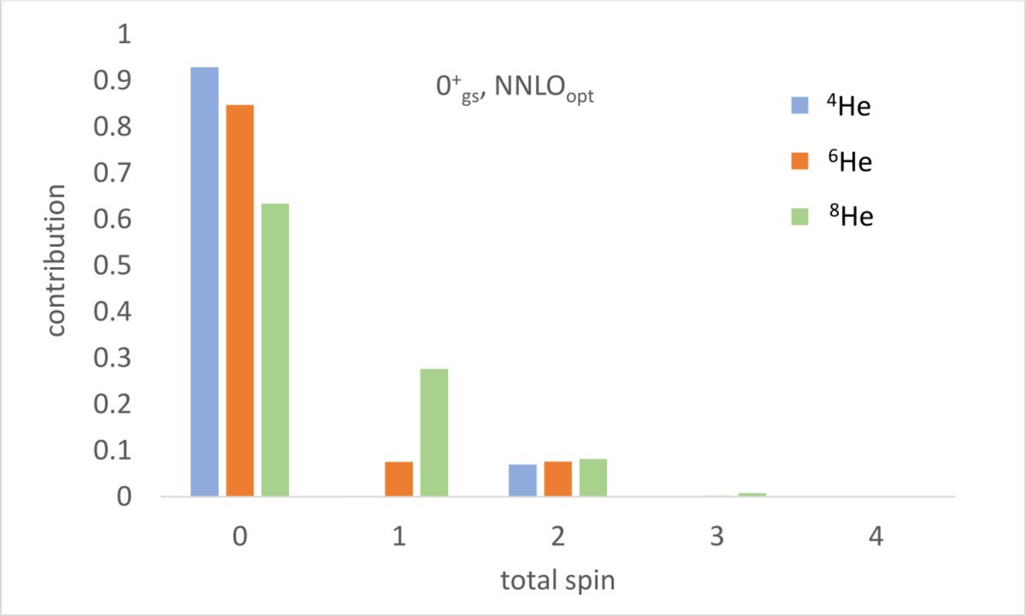




Spin contribution in wave function







p+A and n+A effective interactions (‘optical potentials’)

- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

Today: Consistent approach to p+A effective interaction becomes possible.

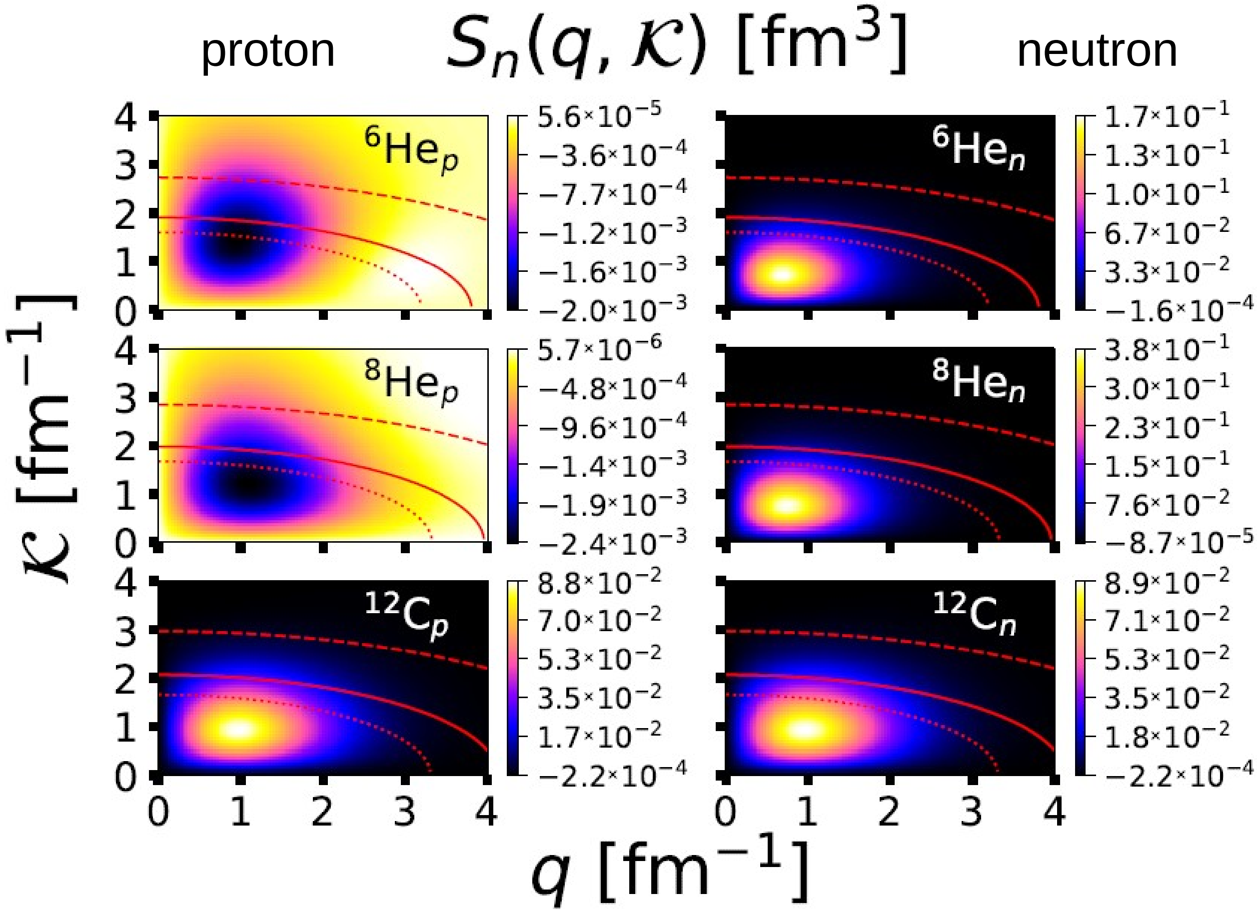
- **In the multiple scattering approach leading order term can be calculated consistently *ab initio***
- **Effect of spin of the struck nucleon visible in spin-observables for N≠Z nuclei in He isotopes**
- Effect in other isotope chains?
  Connection of spin form factors to observables?

- Dependence on NN forces employed
- Refinement of calculation of leading order term for energies below 100 MeV

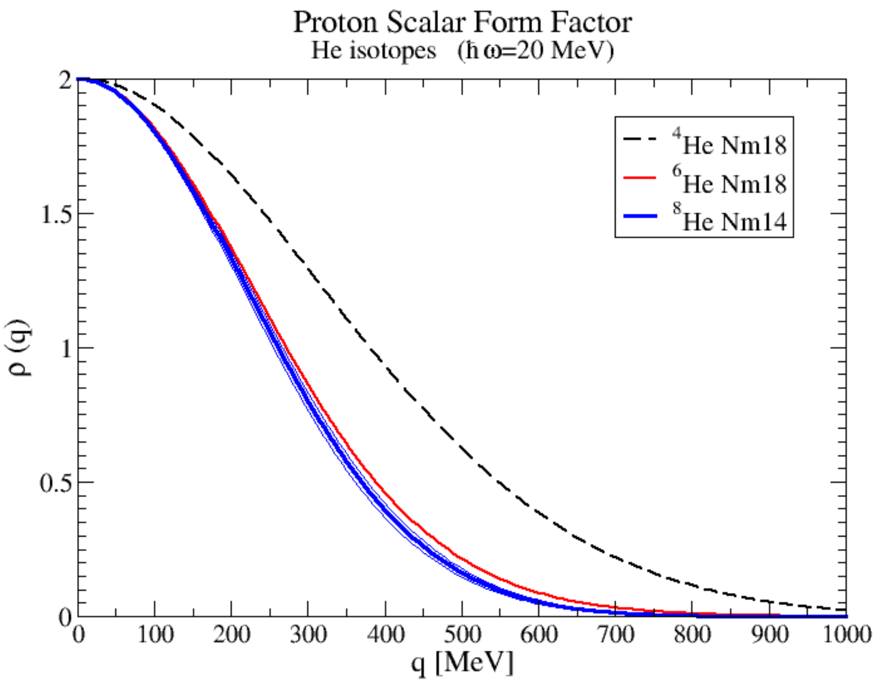
Backup Slides

Off-shell:

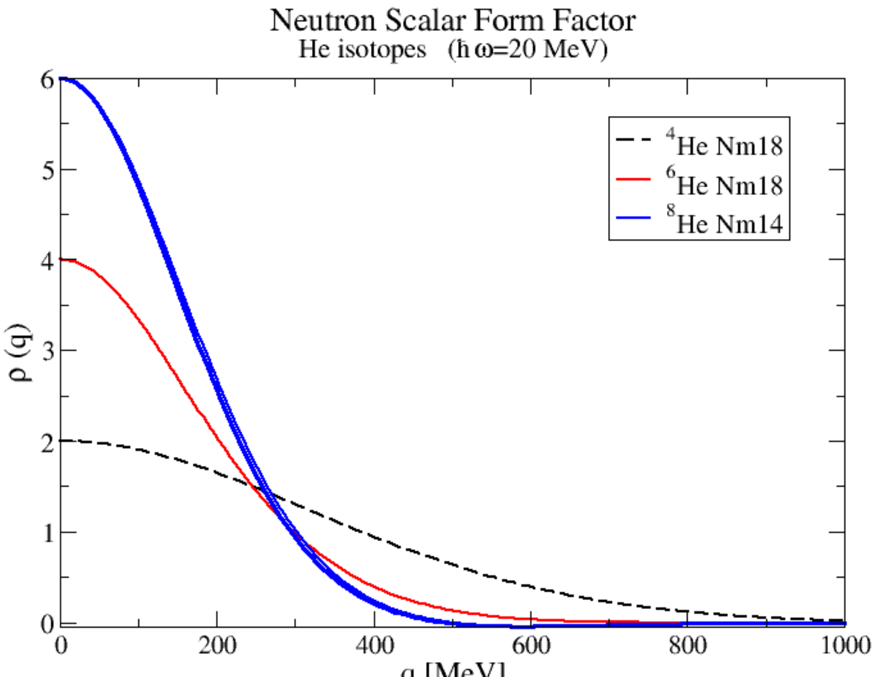
cos (q•K) = 0



Lines indicate on-shell condition: $q^2+4\,K^2 = 4k_0^2$



Local one-body density ρ(q)



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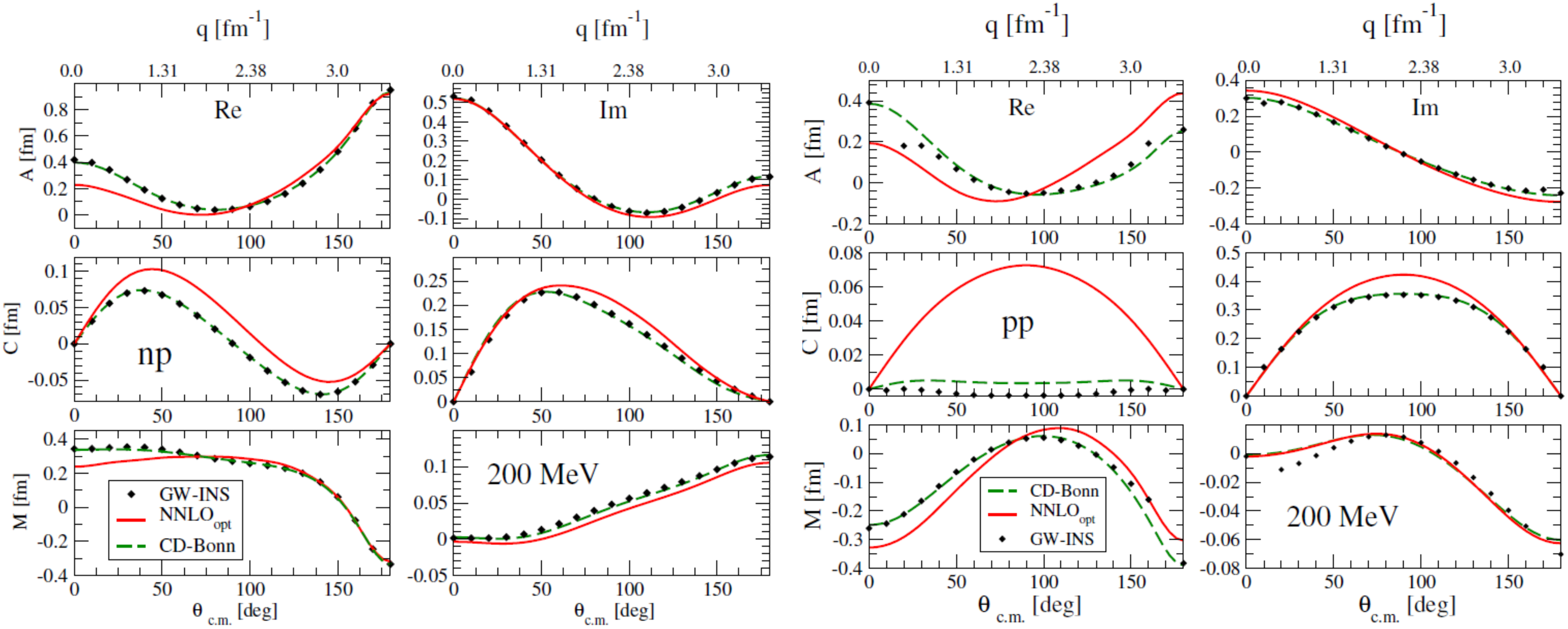
Wolfenstein Amplitudes A, C, M

NNLO$_{opt}$ fitted to E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}

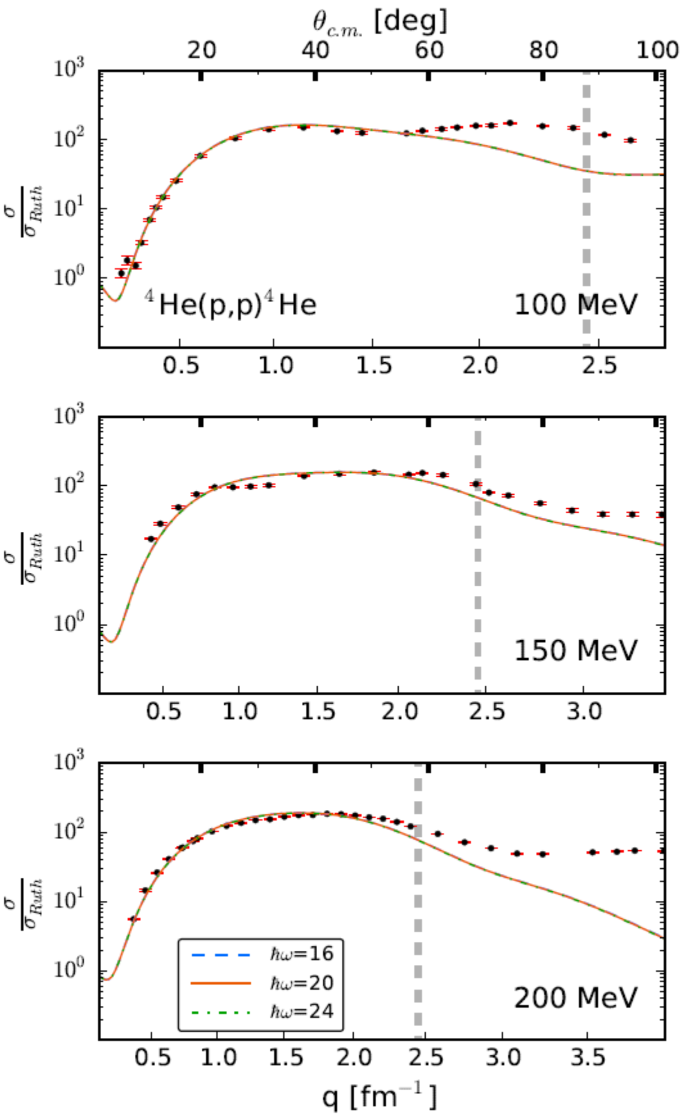


$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

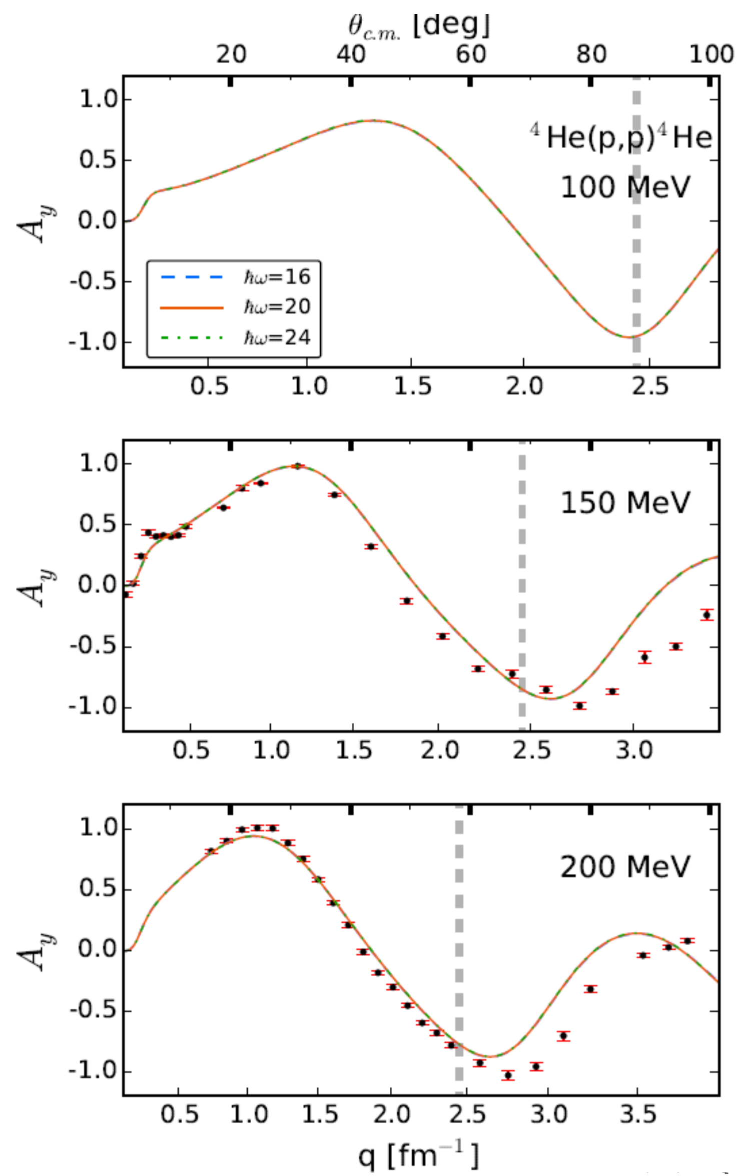
$$q \approx 480\ \text{MeV} = 2.45\ \text{fm}^{-1}$$

Nmax=18

^{4}He



$NNLO_{opt}$ fitted up to Elab=125 MeV



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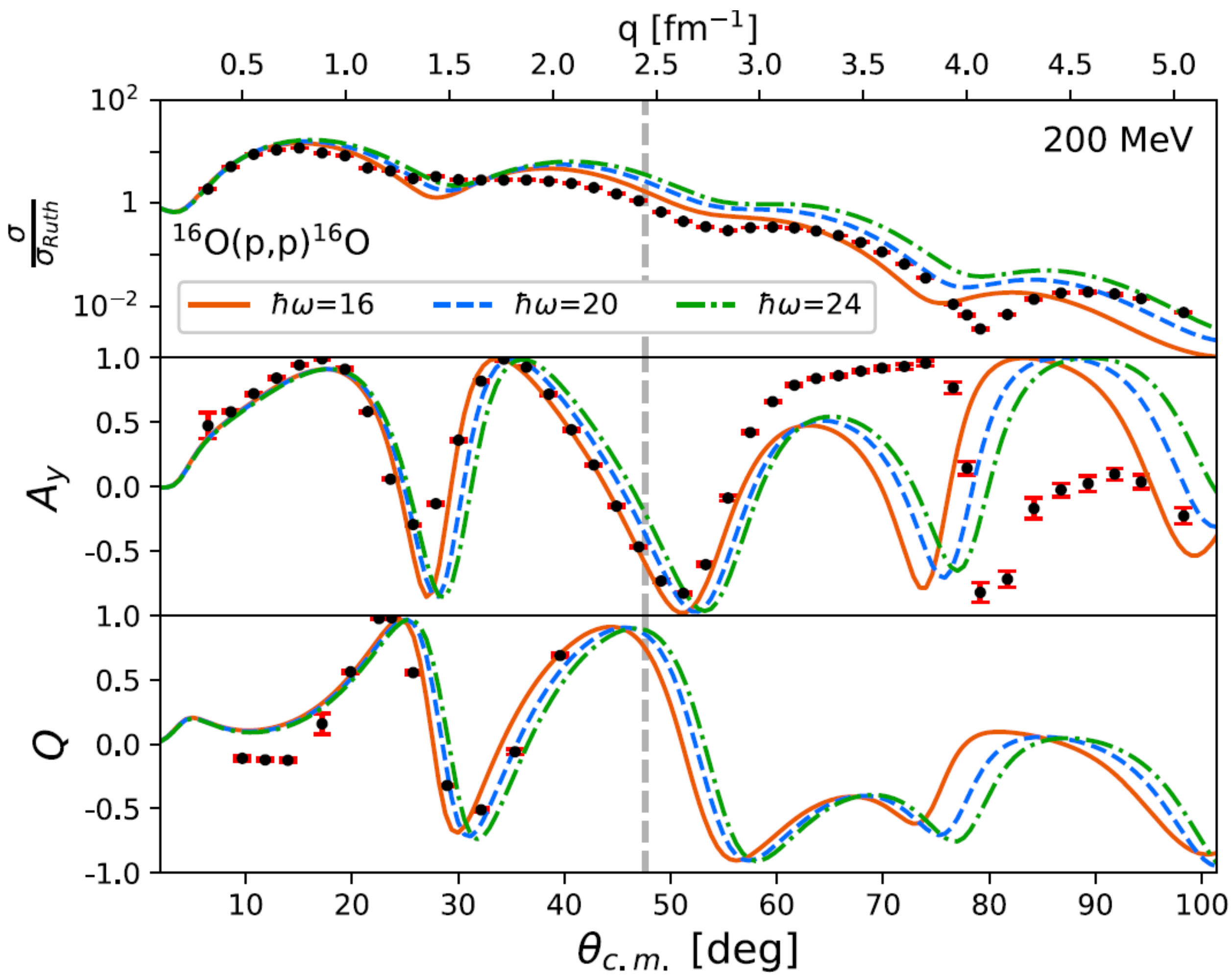
^{16}O

N_{max}=10



$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$

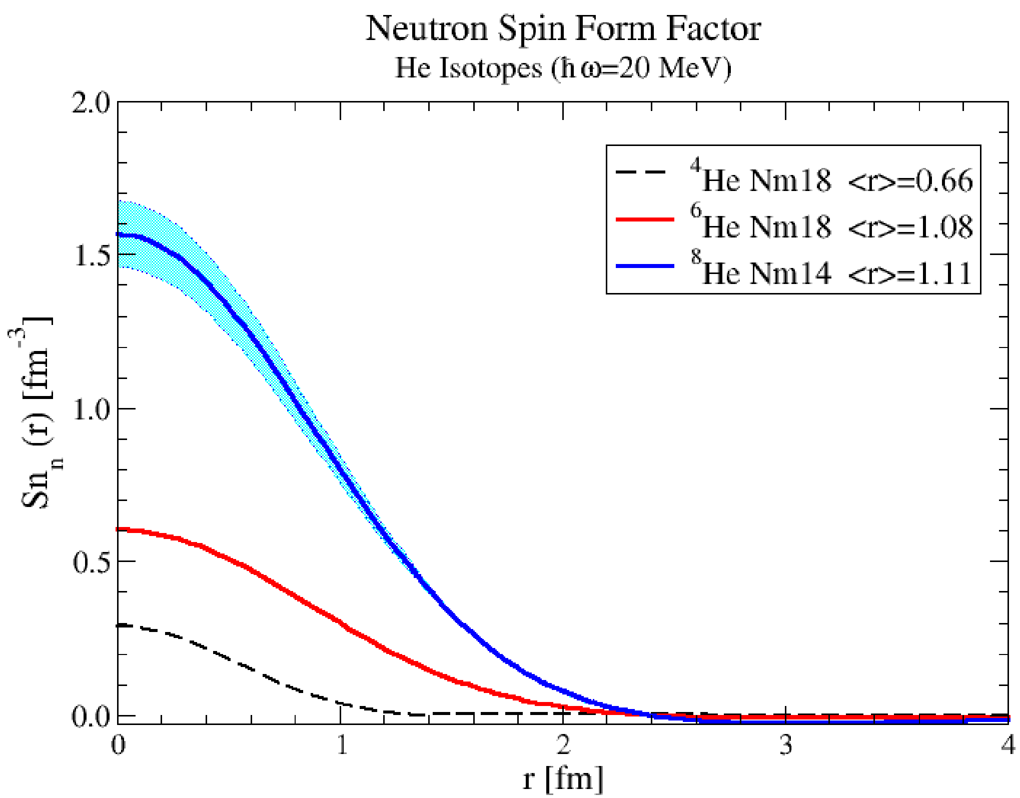
$q \approx 480$ MeV $= 2.45$ fm^{-1}

$NNLO_{opt}$ fitted up to Elab=125 MeV

ħω	Charge Radius
12	2.70318402
16	2.49584345
20	2.37548417
24	2.29262311
Experiment	2.73 ± 0.025 fm[1]

Burrows, Elster, Weppner, Launey, Maris, Nogga, Popa

PHYSICAL REVIEW C **99**, 044603 (2019)



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