Low-energy ^{11}Li + p and ^{11}Li + d scattering in a multicluster model

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1. Introduction
2. Structure of ^{11}Li (E1 distribution)
3. Overview of CDCC (Continuum Discretized Coupled Channels)

   → Systems "3+1", "3+2"
4. The R-matrix method
5. Results on ^{11}Li+p
6. Results on ^{11}Li+d
7. Conclusion

Ref.: P. D., Phys. Rev. C, accepted

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1. Introduction

Two recent experimental papers on ^{11}Li+p and ^{11}Li+d (elastic and inelastic scattering)

- ^{11}Li+p: J. Tanaka et al., Phys. Lett. B 774 (2017) 268
  E_{lab}=66 MeV, E_{cm}=5.5 MeV
  → dipole resonance in ^{11}Li at E_x=0.80 MeV (Γ=1.1 MeV)

- ^{11}Li+d: R. Kanungo et al., PRL 114, 192502 (2015)
  E_{lab}=55.3 MeV, E_{cm}=8.51 MeV
  → dipole resonance in ^{11}Li at E_x=1.03 MeV (Γ=0.5 MeV), isoscalar character

E1 operator

Long wavelength approximation: $M_\mu^{E1} = \sum_i \left(\frac{1}{2} - t_{iz}\right)(\boldsymbol{r}_i - \boldsymbol{R}_{cm})_\mu$

Isoscalar=0 Isovector

Beyond the long wavelength approximation:

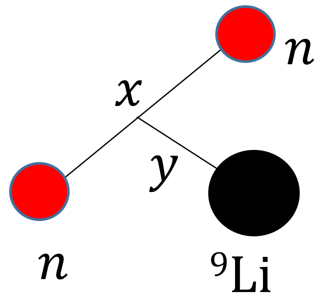
$$M_\mu^{E1} = \sum_i \left(\frac{1}{2} - t_{iz}\right) \boldsymbol{r}'_{i\mu}\left(1 - \frac{1}{10}\left(k_\gamma r'_i\right)^2 + \cdots\right) + \cdots$$

➔ isoscalar transitions are possible

1. Introduction

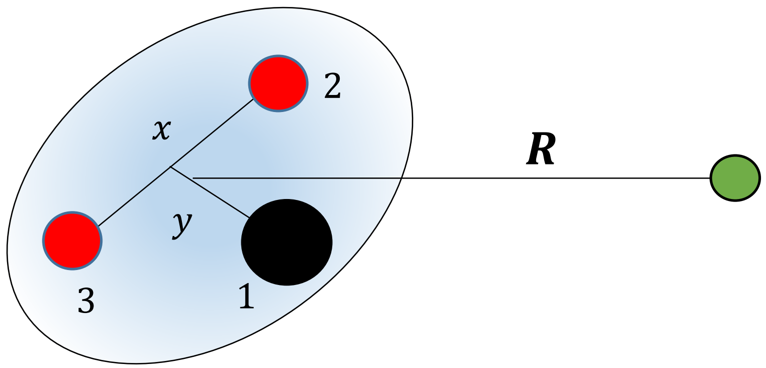
Present work

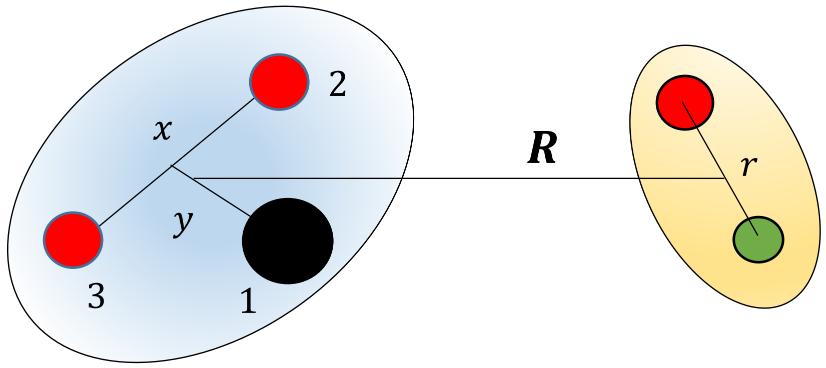
- ^{11}Li=^{9}Li+n+n (hyperspherical coordinates): E. C. Pinilla, P. Descouvemont, and D. Baye, Phys. Rev. C 85, 054610 (2012).



➔ E1 distribution

- ^{11}Li+p, ^{11}Li+d with the CDCC method



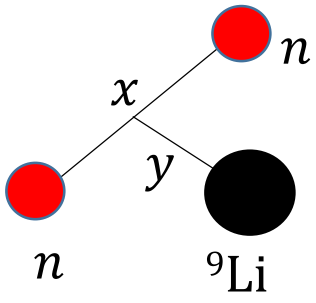


^{11}Li+p: 4-body CDCC (3+1)
(see also T. Matsumoto et al., PTEP 2019, 126)

^{11}Li+d: 5-body CDCC (3+2)

2. Structure of ^{11}Li

2. Structure of ^{11}Li

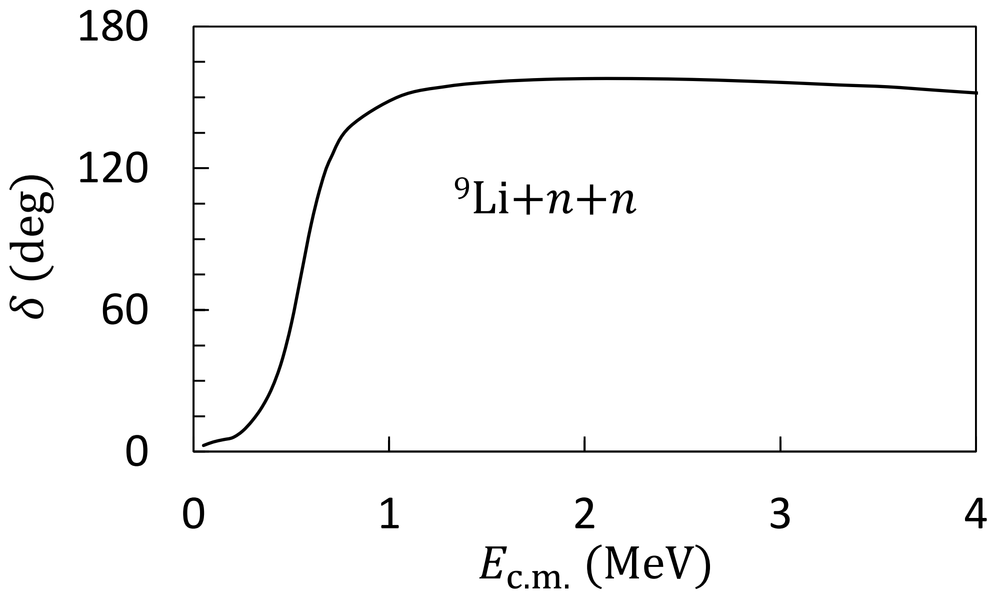


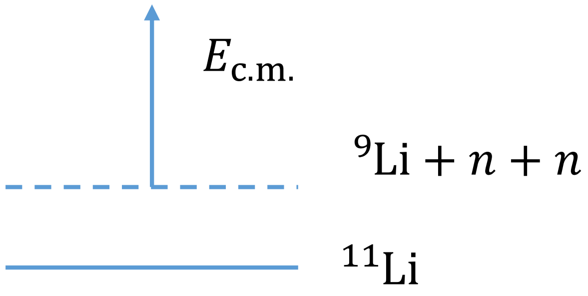
- Details in E. C. Pinilla et al., Phys. Rev. C 85, 054610 (2012).
- V_{nn}=Minnesota potential
- V_{9Li+n}=Woods-Saxon fitted on the scattering length
- Spin of the ^{9}Li core is neglected
- Forbidden states for $s_{1/2}$ and $p_{3/2}$ → removed by a supersymmetric transformation

J=0^{+}

- Bound state at E_B=-0.378 MeV
- $\sqrt{<r^2>}$=3.12 fm, exp=3.16±0.11 fm

J=1^{-}: ^{9}Li+n+n phase shifts (3-body phase shifts)





➔ Dipole resonance near E_{cm}=0.6 MeV, E_x=1.0 MeV

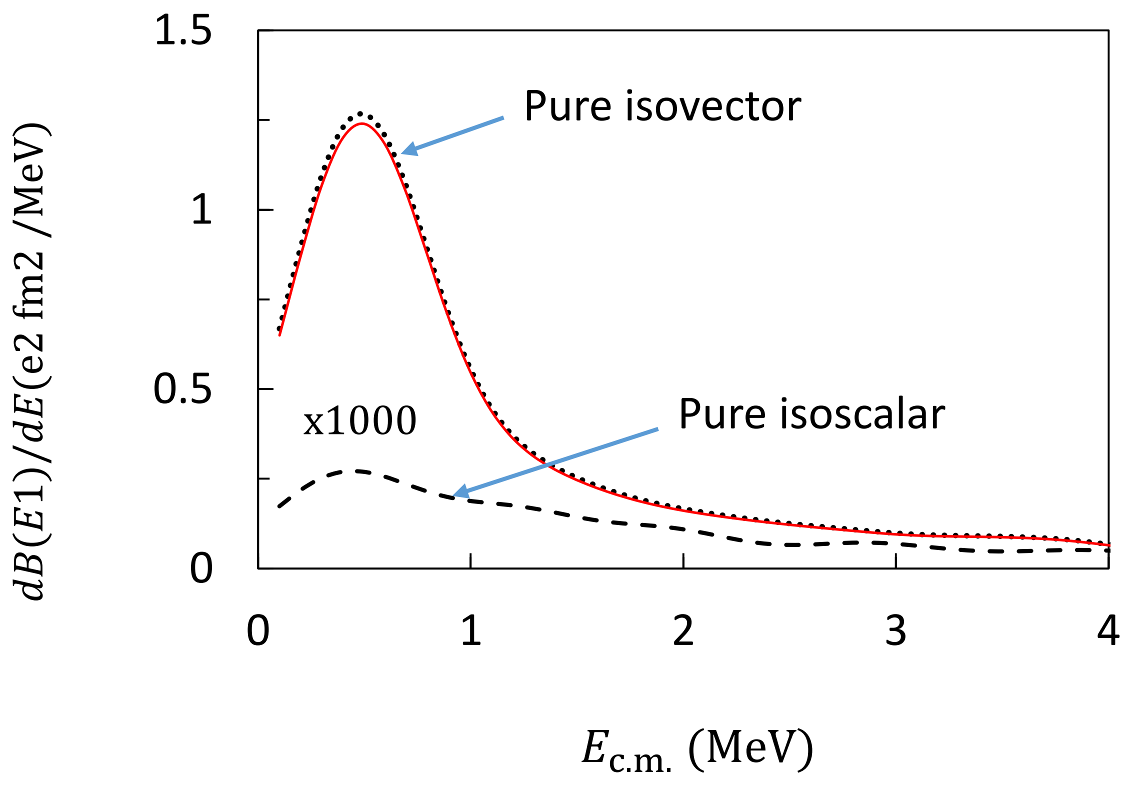
2. Structure of ^{11}Li

E1 transitions

$$B(E1, J_i \to J_f) = \frac{2J_f+1}{2J_i+1}\left|< \Psi^{J_f} \| M^{E1} \| \Psi^{J_i} >\right|^2$$

2 options for M_μ^{E1}: LWA → isoscalar=0

beyond the LWA → isoscalar ≠0



→ Peak near E_{cm}=0.6 MeV
Consistent with the phase shifts

→ Weak influence of high-order terms in M_μ^{E1}
Term ~$\frac{1}{10}(k_\gamma r)^2$ with $k_\gamma = (E_{cm} + 0.4)/\hbar c$
Even if r^2 is large the correction is quite small

→ No isoscalar character for the transition

Overview of CDCC :

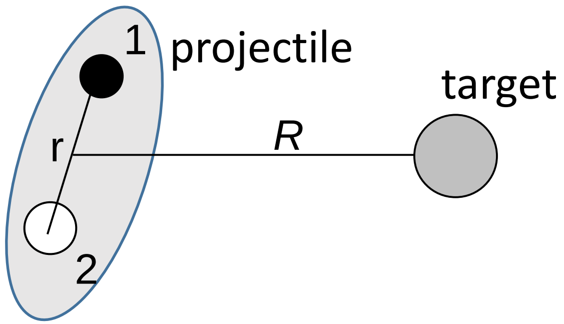
<u>C</u>ontinuum <u>D</u>iscretized <u>C</u>oupled <u>C</u>hannel method

3. Overview of CDCC

- Introduced in the 70's to deal with deuteron scattering
  Low binding energy of the deuteron → breakup is important
    - G. Rawitscher, Phys. Rev. C 9, 2210 (1974)
    - N. Austern et al., Phys. Rep. 154 (1987) 126

- Two-body projectile, three-body problem

$$H = H_0(\boldsymbol{r}) - \frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{R}} + V_{t1}(\boldsymbol{R},\boldsymbol{r}) + V_{t2}(\boldsymbol{R},\boldsymbol{r})$$



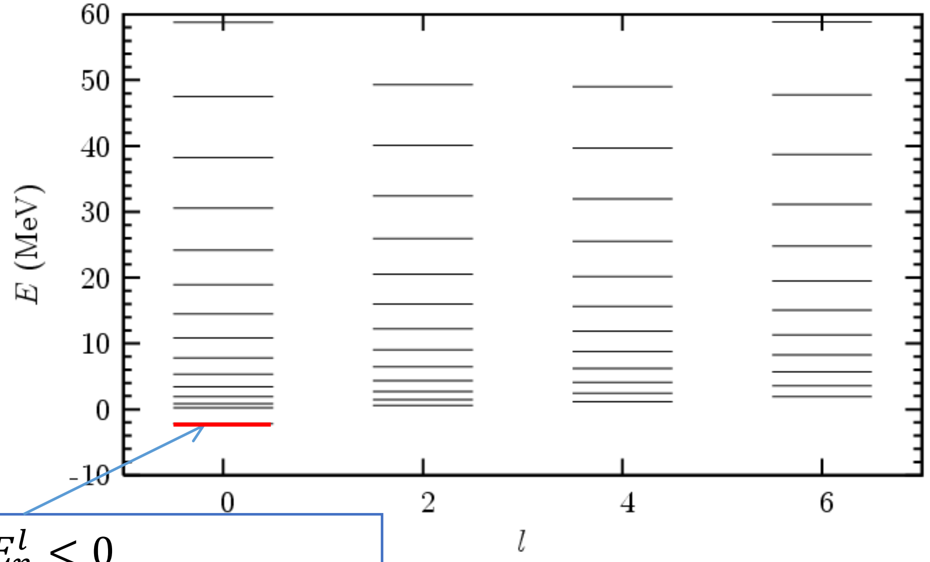
- $H_0(\boldsymbol{r})$ = Hamiltonian associated with the projectile

- V_{t1}, V_{t2}= optical potentials between the target and the fragments
  (high energies: above the resonance region)

3. Overview of CDCC

- Projectile breakup described by approximate (discrete) states: $H_0\Phi_n^{lm}(\boldsymbol{r}) = E_n^l\Phi_n^{lm}(\boldsymbol{r})$

Example: d=p+n



Pseudo state (PS): $E_n^l > 0$
- Simulate breakup effects
- No physical meaning
- Depend on the basis

Ground state: $E_n^l < 0$
- Physical
- Does not depend on the basis

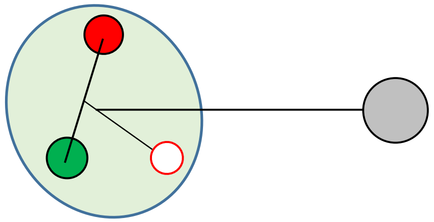
- CDCC well adapted to exotic nuclei (low binding energy)
  Example: $^{11}Be = ^{10}Be + n$ (0.5 MeV)

- Low BU energy is not necessary! But BU effects are expected to be more important

3. Overview of CDCC

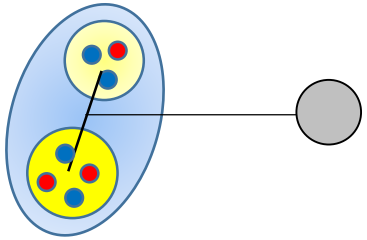
Extensions: same principle : discretzation of the continuum

- 3-body projectiles: ^{6}He, ^{11}Li, ^{9}Be



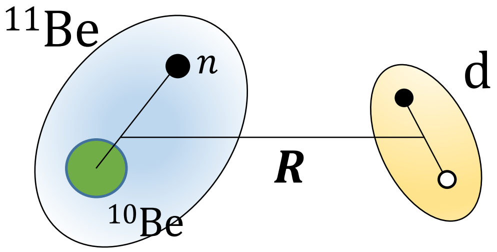
- T. Matsumoto et al., PRC70 (2004) 061601
- M. Rodriguez-Gallardo et al., PRC77 (2008) 064609

- A-body projectiles: ^{7}Li, ^{6}He, ^{8}Li



Based on nucleon-target potentials → no parameter
- Y. Sakuragi et al., PTP Supp. 89 (1986) 136
- P.D., M. Hussein, PRL 111 (2013) 082701

- 2-body projectile + 2-body target: ^{11}Be+d, ^{7}Li+d

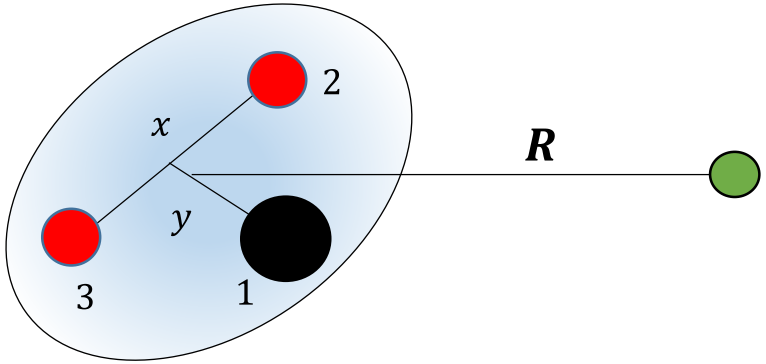


Pseudostates in the projectile and in the target → many channels
- P. D., Phys. Lett. B 772 (2017) 1
- P. D., Phys. Rev. C 97 (2018) 064607

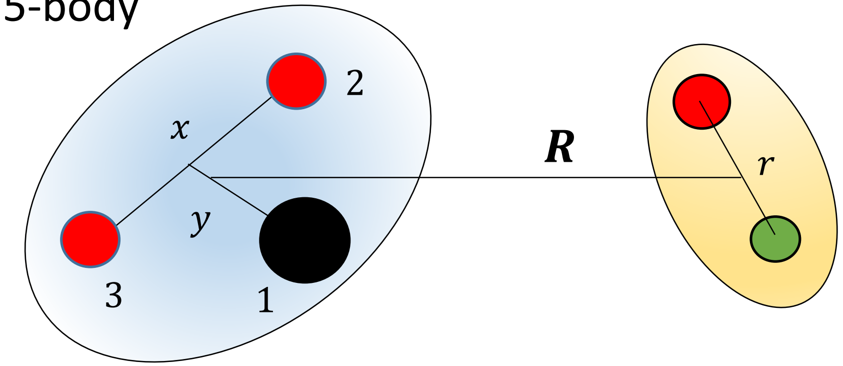
3. Overview of CDCC

- CDCC equations for ^{11}Li+p and ^{11}Li+d

^{11}Li+p: 4-body



^{11}Li+d: 5-body



Total hamiltonian: $H = H_1(\boldsymbol{x},\boldsymbol{y}) + H_2(\boldsymbol{r}) + T_R + \sum_{ij} U_{ij}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y},\boldsymbol{r})$

With H_i=internal hamiltonian of nucleus i
T_R =relative kinetic energy
$U_{ij}(\boldsymbol{s})$ =optical potential between fragments i and j

Then: standard CDCC procedure

3. Overview of CDCC

Standard CDCC procedure:

1. Step 1: solve $H_1\Phi_{1k}^{jm} = E_{1k}^{j}\Phi_{1k}^{jm}$ for ^{11}Li (hyperspherical coordinates)
   $H_2\Phi_{2k}^{jm} = E_{2k}^{j}\Phi_{2k}^{jm}$ for d

With Φ_{1k}^{jm} expanded on a basis (Lagrange functions: matrix elements are simple)

→ negative energies = physical states
positive energies = pseudostates=(discrete) approximations of the continuum in 1 and 2

2. Step 2:

Define channel functions: $\varphi_c(\boldsymbol{x},\boldsymbol{y},\boldsymbol{r},\Omega_R) = \left[\left[\underbrace{\Phi_{1k_1}^{j_1}(\boldsymbol{x},\boldsymbol{y})}_{\boxed{^{11}\mathrm{Li}}} \otimes \underbrace{\Phi_{2k_2}^{j_2}(\boldsymbol{r})}_{\boxed{\mathrm{d}}}\right]^{I} \otimes Y_L(\Omega_R)\right]^{JM}$

with
I = channel spin
L =angular momentum between d and ^{11}Li
index $c = (j_1, k_1, j_2, k_2, I, L)$

and expand the total wave function as $\Psi^{JM\pi} = \sum_c u_c^{J\pi}(R)\varphi_c(\boldsymbol{x},\boldsymbol{y},\boldsymbol{r},\Omega_R)$

$u_c^{J\pi}(R)$ to be determined

3. Overview of CDCC

3. Step 3

Compute matrix elements of the potential $\sum_{ij} U_{ij}(\boldsymbol{R}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{r})$

$$V^{J}_{cc'}(R) = \langle \varphi_c \left| \sum_{ij} U_{ij}(\boldsymbol{R}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{r}) \right| \varphi_{c'} \rangle$$

= integrals over 11 coordinates (8 angles + 3 radii): 5 analytical + 6 numerical integrals
(use of the Raynal-Revai coefficients)

4. Step 4: Solve the coupled-channel system

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+E_c-E\right]u_c^{J\pi}(R)+\sum_{c'}V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R)=0$$

- Standard coupled-channel system (general form common to most scattering theories)
- At large distances (only Coulomb) : $u_c^{J\pi}(R)\to I_c(R)\delta_{c\omega}-O_c(R)U_{c\omega}^{J\pi}$ (ω =entrance channel)
  $U_{c\omega}^{J\pi}$ = scattering matrix: provides the cross sections (elastic, inelastic, breakup, etc.)
- Solved with the R-matrix method (space divided in an internal and an external regions)
- The system must be solved for each $J\pi$
- Problems:
  - Many channels c (up to 9000 for ^{11}Li+d)
  - Many $J\pi$ values (depends on energy)
  - Long range of the potentials $V_{cc'}^{J\pi}(R)$ (due to Coulomb)

  ➔ Long calculations + many tests

5. Step 5
   Determing the cross sections from the scattering matrices

The R-matrix method

4. The R-matrix method

Scattering matrix determined from the R-matrix theory

R-matrix theory: based on 2 regions (channel radius a)

Lane and Thomas, Rev. Mod. Phys. 30 (1958) 257
P.D. and D. Baye, Rep. Prog. Phys. 73 (2010) 036301
P.D., Computer Physics Communications 200 (2016) 199

Internal region: $R \leq a$ $\qquad$ $R = a$ $\qquad$ External region: $R \geq a$ $\qquad$ R

Full Hamiltonian

$u_c^{J\pi}(R)$ expanded over a basis (N functions $\phi_i(R)$)

$$\boldsymbol{u_{c,int}^{J\pi}(R)} = \sum_{i=1}^{N} c_i \phi_i(R)$$

$\phi_i(R)$=Lagrange functions typically N ~50

Only Coulomb (monopole)

$$V_{cc'}^{J\pi}(R) = \frac{Z_p Z_t e^2}{R} \delta_{cc'}$$

$u_c^{J\pi}(R)$ has its asymptotic form

$$\boldsymbol{u_{c,ext}^{J\pi}(R)} = I_c(kR)\delta_{c\omega} - O_c(kR) U_{c\omega}^{J\pi}$$

- Main ingredient: matrix elements of the coupling potentials $V_{cc'}^{J\pi}(R)$: $< \phi_i | V_{cc'}^{J\pi} | \phi_j >$ → fast method needed
- Matching at *R=a* provides: scattering matrices $\boldsymbol{U^{J\pi}}$ → cross sections

4. The R-matrix method

Choice of the basis: **the Lagrange-mesh method** (*D. Baye, Phys. Rep. 565 (2015) 1-107)*

- Gauss approximation: $\int_0^a g(x)dx \approx \sum_{k=1}^{N} \lambda_k g(x_k)$
  - N= order of the Gauss approximation
  - x_k=roots of an orthogonal polynomial $P_N(x)$, λ_k=weights
  - If interval [0,*a*]: Legendre polynomials
[0,∞]: Laguerre polynomials

- Lagrange functions for [0,1]: $f_i(x) \sim \frac{P_N(2x-1)}{(x-x_i)}$
  - x_i are roots of $P_N(2x-1)=0$
  - with the Lagrange property: $f_i(x_j) = \lambda_i^{-1/2}\delta_{ij}$

- Matrix elements with Lagrange functions: Gauss approximation is used

$< f_i|f_j > = \int f_i(x)f_j(x)dx \approx \delta_{ij}$

$< f_i|T|f_j >$ analytical

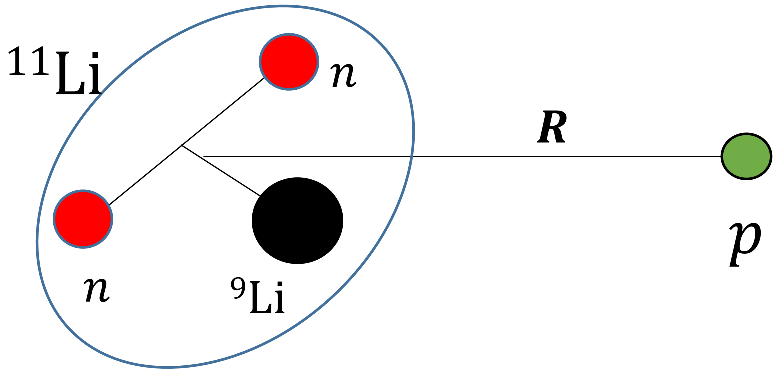
$< f_i|V|f_j > = \int f_i(x)V(x)f_j(x)dx \approx V(x_i)\delta_{ij}$ $\Rightarrow$ **no integral needed**

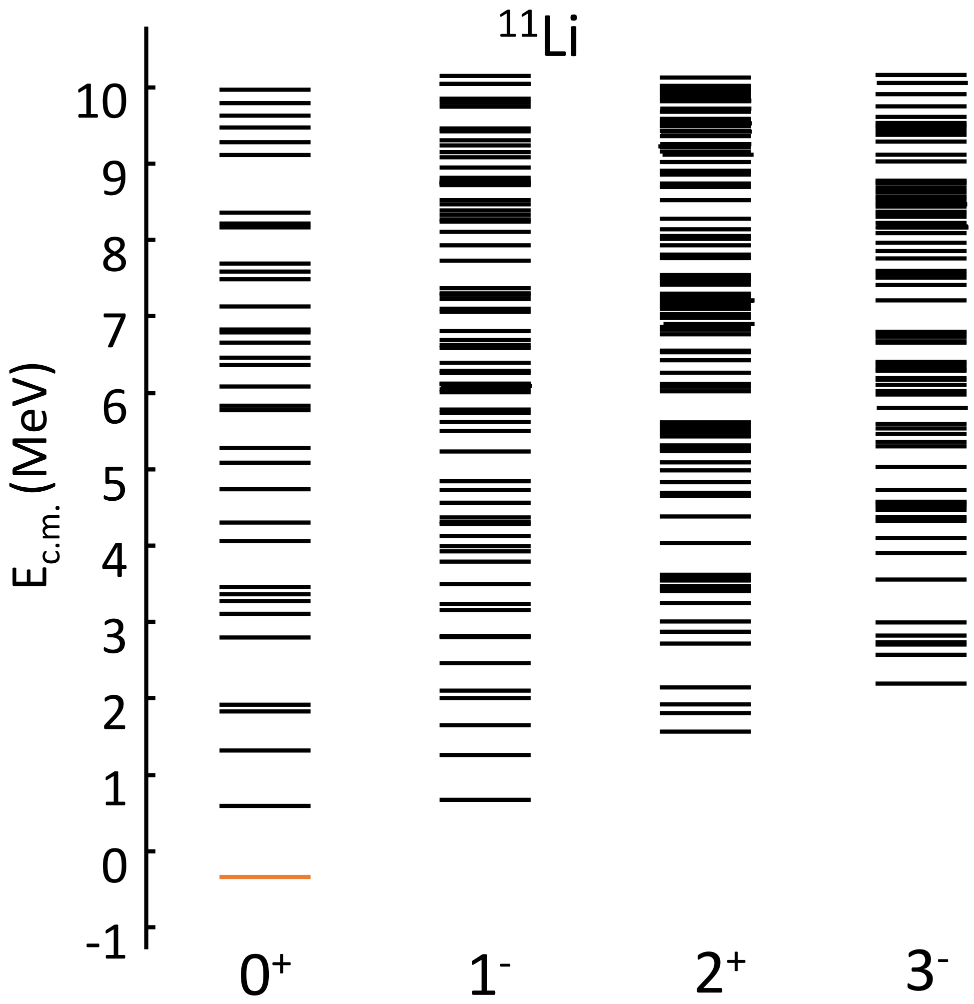
Also applicable to non-local potentials

Results on ^{11}Li+p

5. Results on ^{11}Li+p

a. Conditions of the calculations





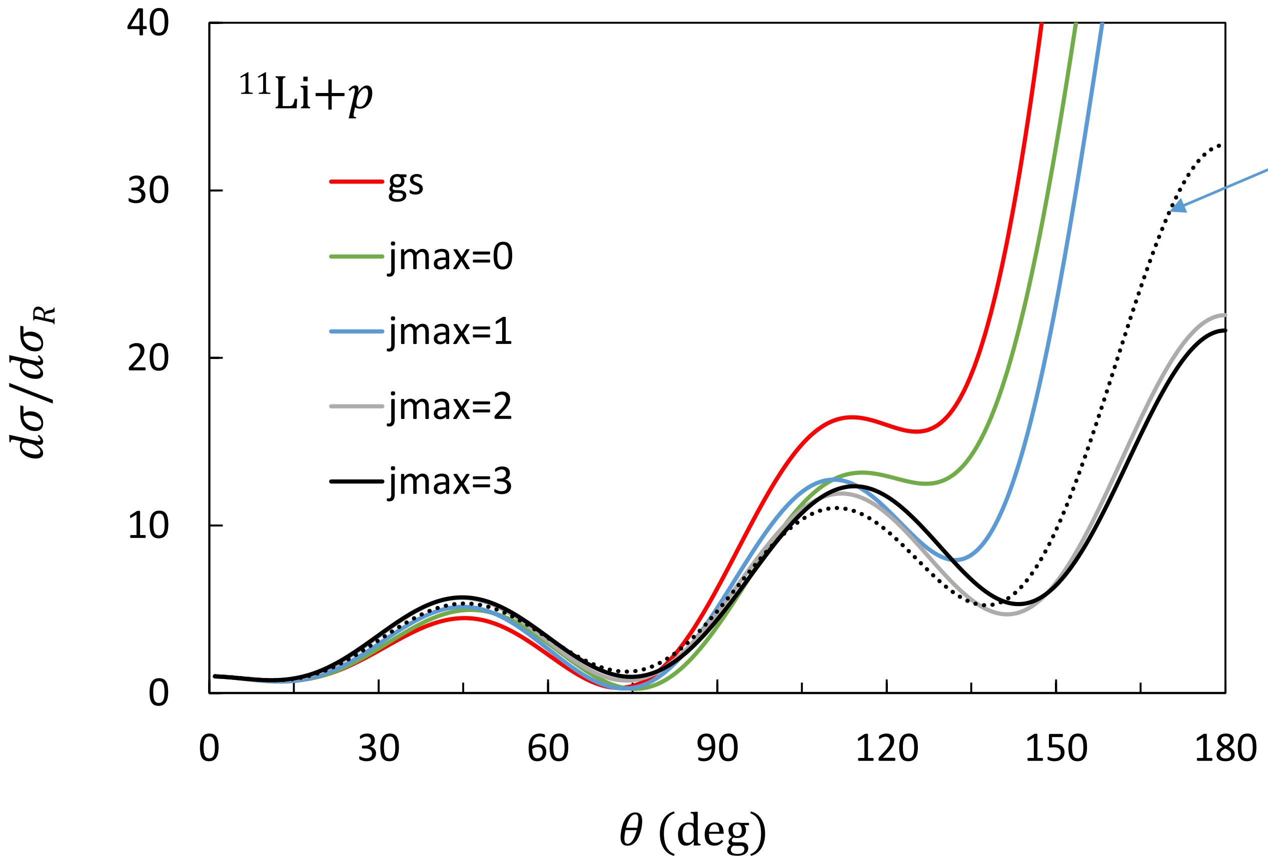
Interactions
- n+p: Minnesota
- ^{9}Li+p: Koning-Delaroche, Chapel Hill

Channel radius
a~25 fm (stability tests)

^{11}Li pseudostates
E_{max}=10 MeV, j_{max}=3

5. Results on ^{11}Li+p

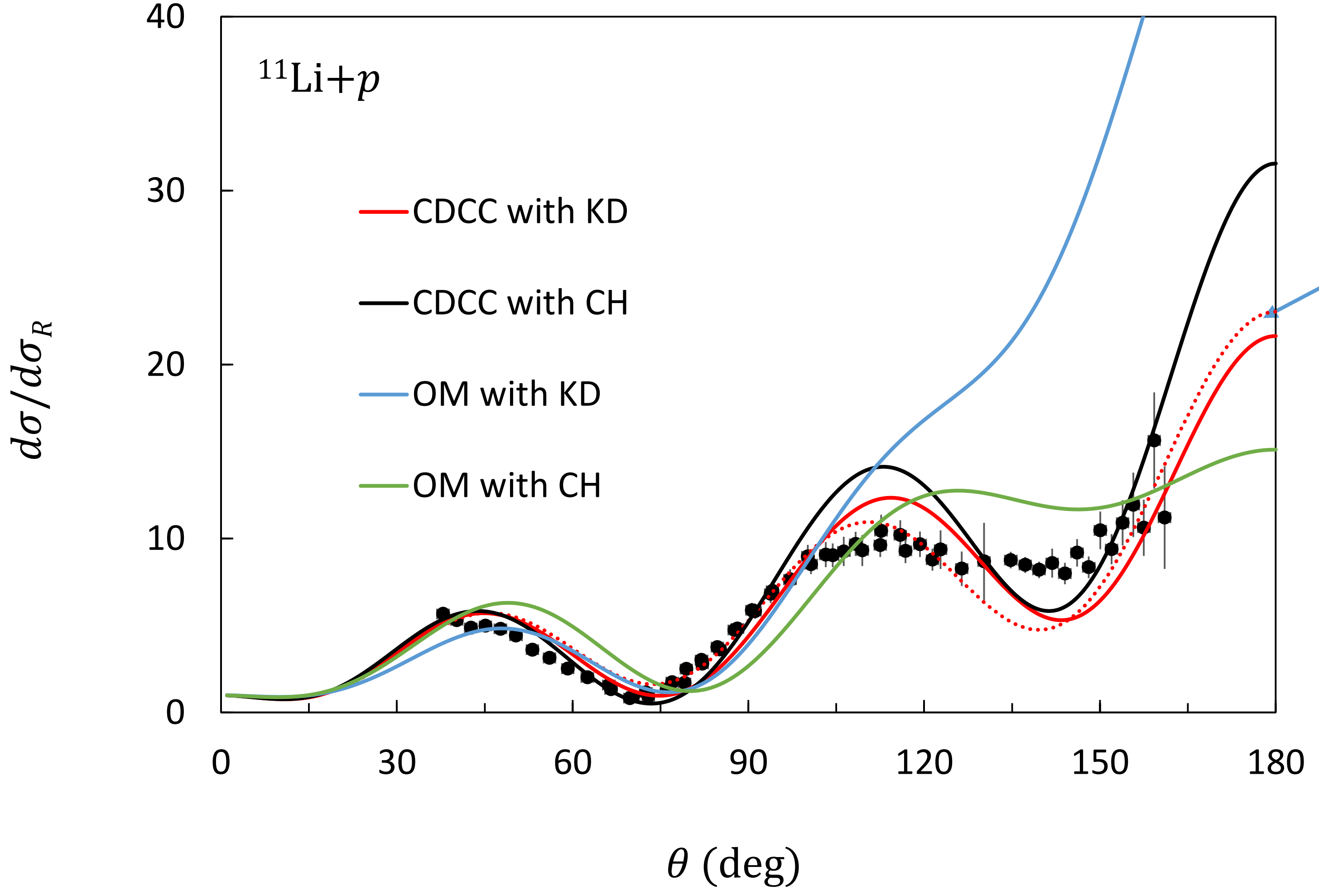
b. Convergence of the elastic cross section, E_{lab}=66 MeV, E_{cm}=5.5 MeV



PS up to Emax=5.5 MeV
→ Closed channels are neglected

c. Comparison with experiment

OM: optical model with global parametrizations (KD03, CH89)



Smaller basis for ^{11}Li
Will be used for ^{11}Li+d

5. Results on ^{11}Li+p

d. Equivalent potentials

Question: can we find a single-channel equivalent potential?

- **J-dependent potential**

For the elastic channel : $\left(T_R + V_{11}^J(R) - E\right)u_1^J(R) = -\sum_{c\neq 1} V_{1c}^J(R)u_c^J(R)$

Equivalent to $\left(T_R + V_{11}^J(R) + V_{pol}^J(R) - E\right)u_1^J(R) = 0$

with $V_{pol}^J(R) = -\frac{\sum_{c\neq 1} V_{1c}^J(R)u_c^J(R)}{u_1^J(R)}$

Problems: J dependent
contains singularities (nodes of the wave function)

→ Construction of a J-independent potential

5. Results on ^{11}Li+p

b) J-independent potential

I.J. Thompson et al., Nucl. Phys. A 505 (1989) 84.

$$V_{pol}(R) = \frac{\sum_J V_{pol}^J(R)\omega^J(R)}{\sum_J \omega^J(R)}$$

With $\omega^J(R)$=weight function

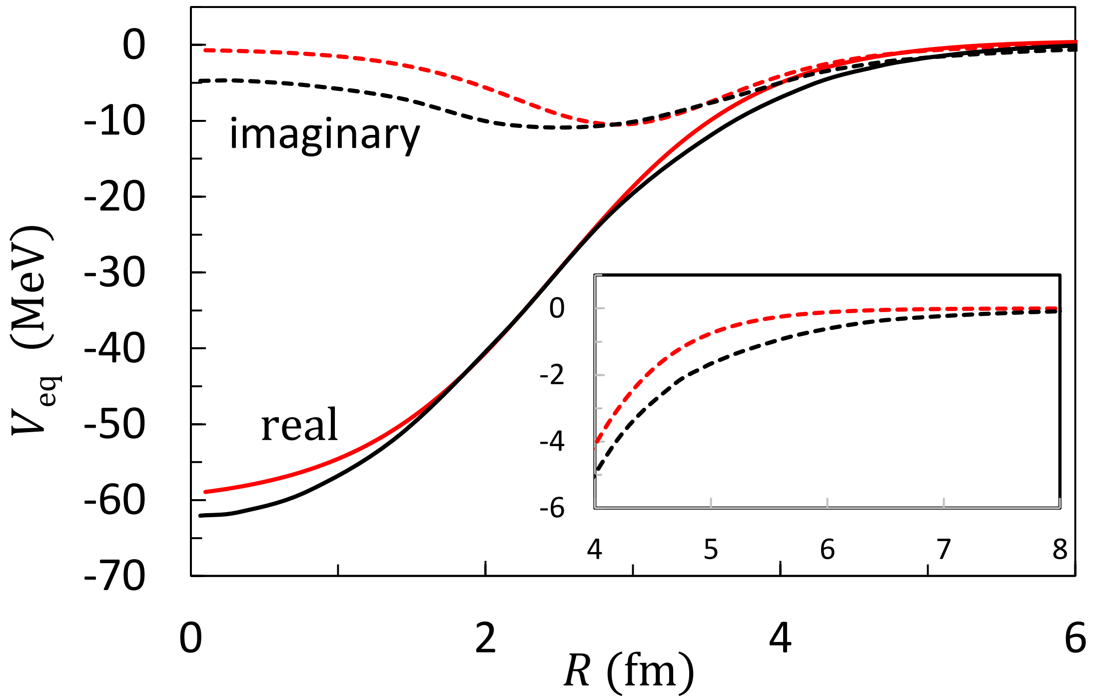
$$\omega^J(R) = (2J+1)\left(1 - \left|U_{11}^J\right|^2\right)\left|u_1^J(R)\right|^2$$

reduces the influence of the nodes

gives more weight to the dominant J-values

Test: verify that $V_{pol}(R)$ redroduces the full calculation

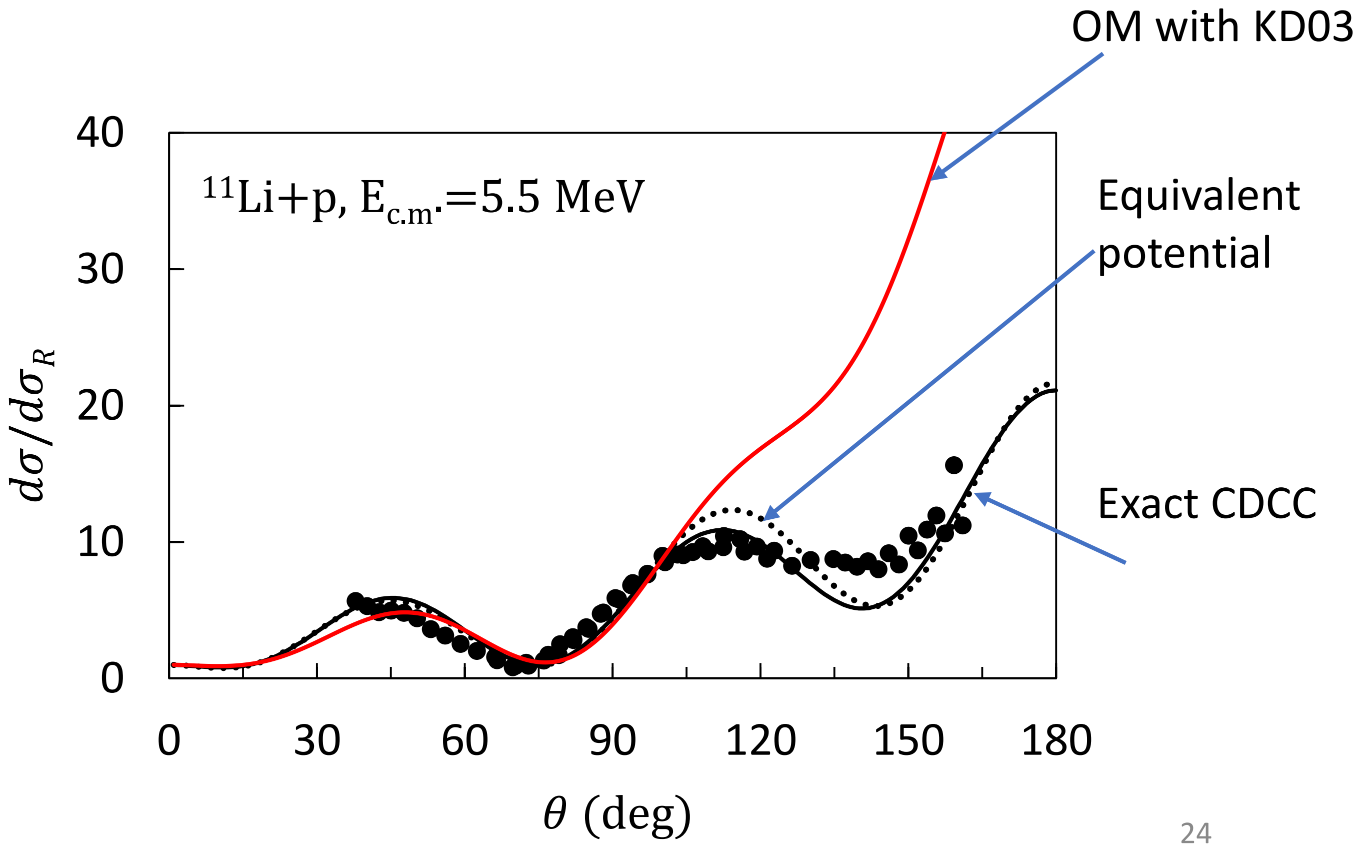
5. Results on ^{11}Li+p



Black: CDCC equivalent potential

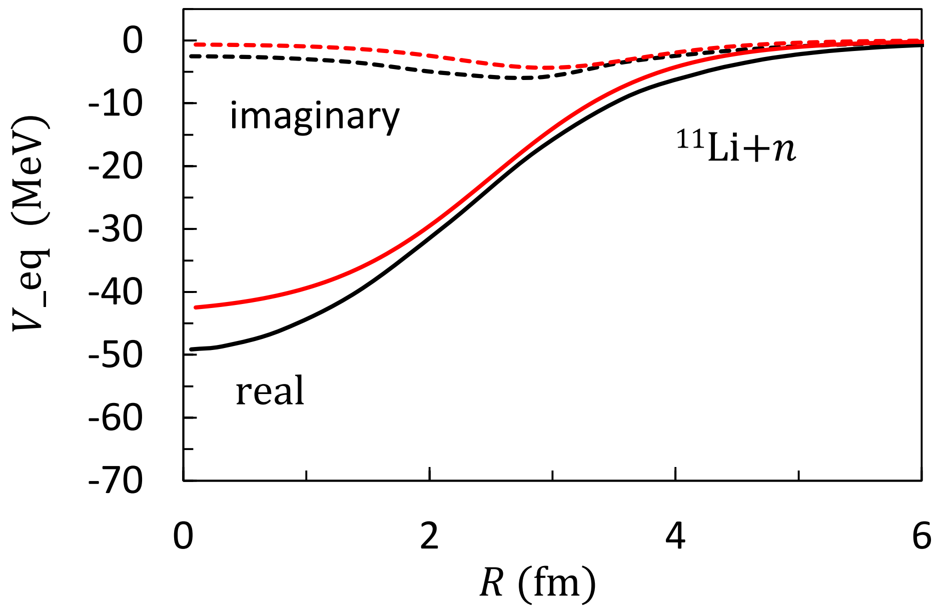
Red: KD ^{11}Li+p potential

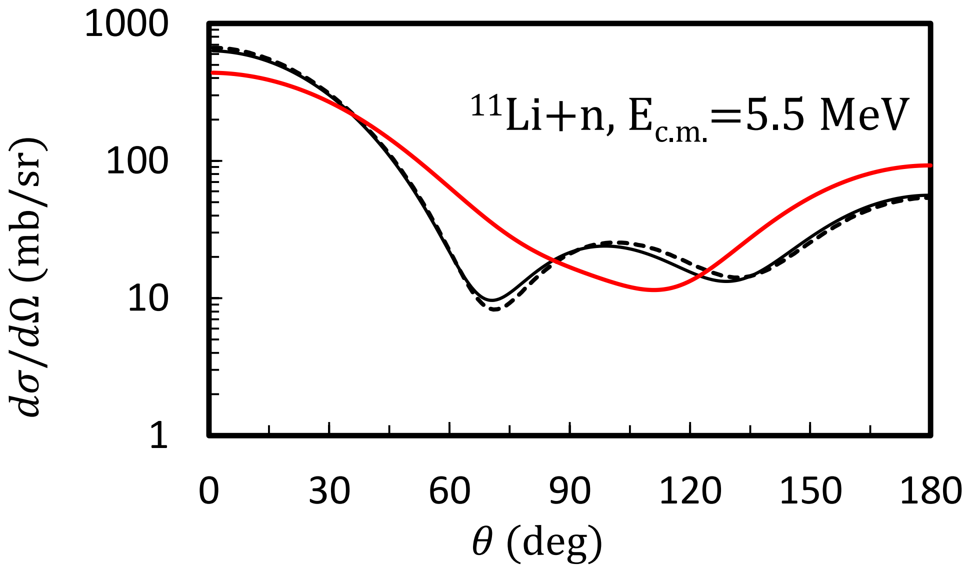
→ Range larger for CDCC



5. Results on ^{11}Li+p

^{11}Li+n potential (used for ^{11}Li+d)



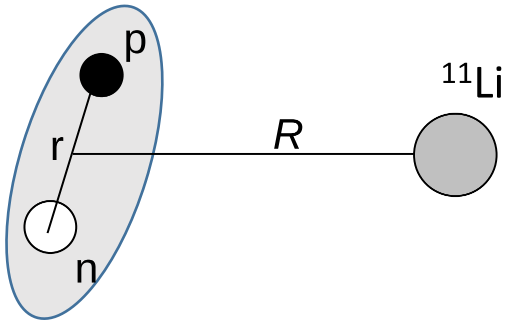


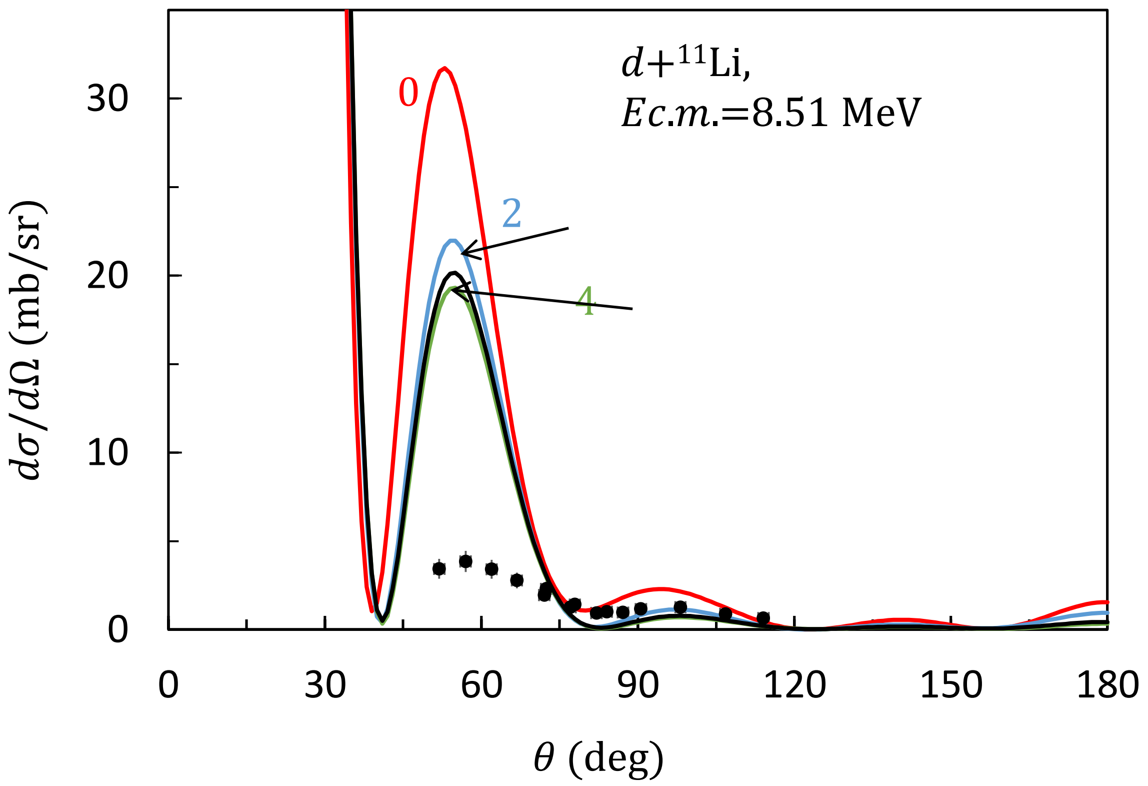
Results on ^{11}Li+d

6. Results on ^{11}Li+d

- Main goal: simultaneous study of ^{11}Li+p and ^{11}Li+d (same conditions)
- Much more difficult: many channels, coupling potentials require long computer times, etc
  → no full convergence

First calculation: « standard » CDCC calculation with ^{11}Li+p/n equivalent potentials (deuteron BU only)

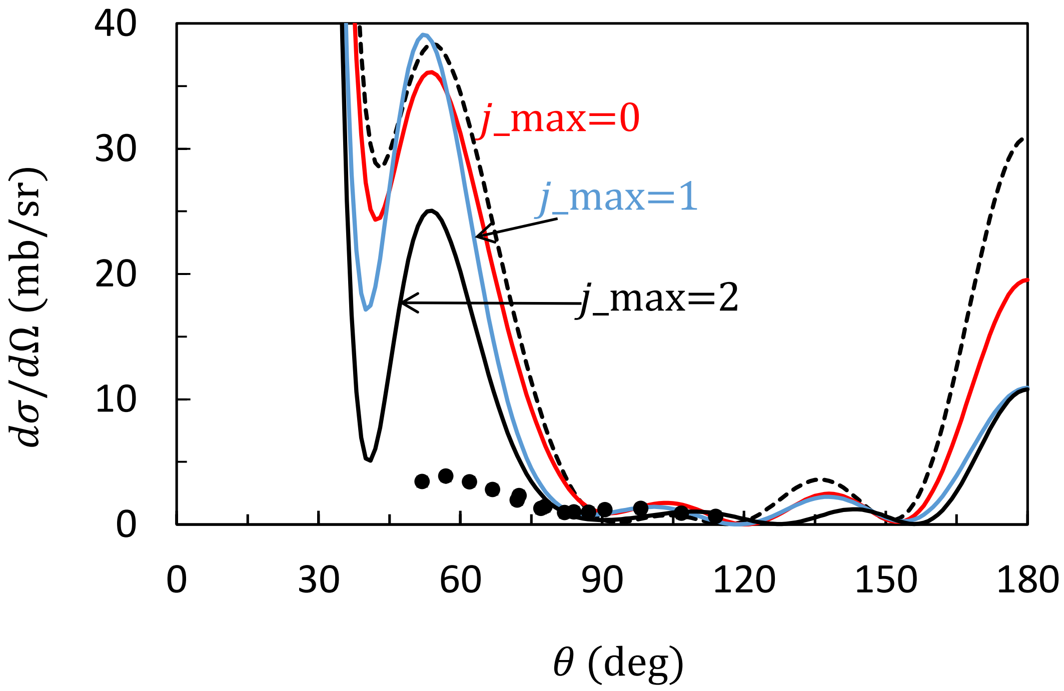




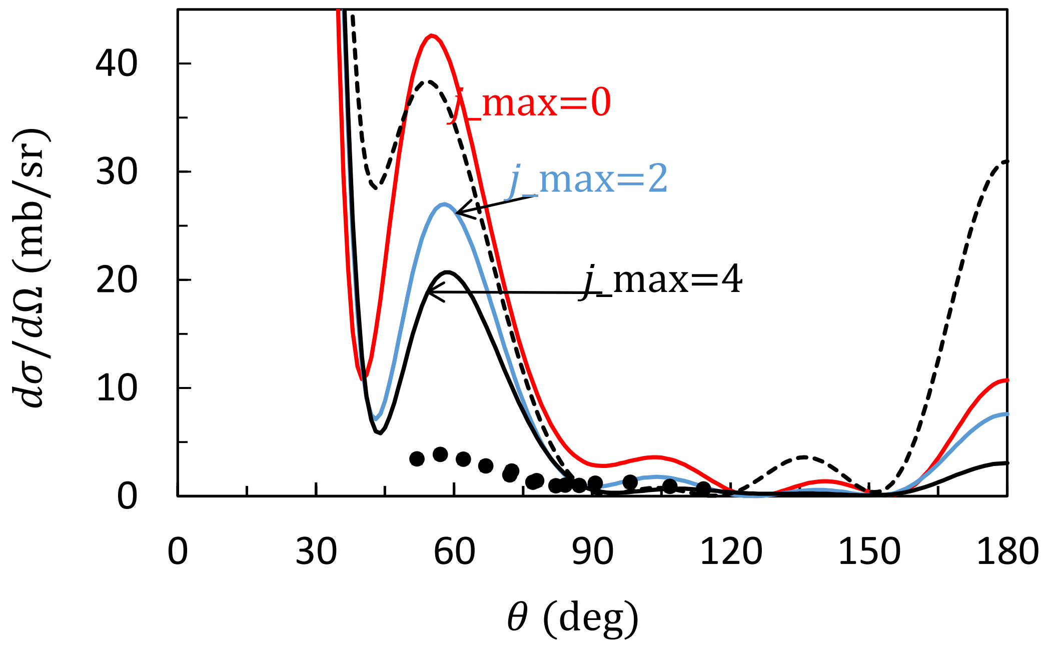
6. Results on ^{11}Li+d

Second calculation: 5-body CDCC calculation

Convergence with respect to ^{11}Li BU
(d in ground state)



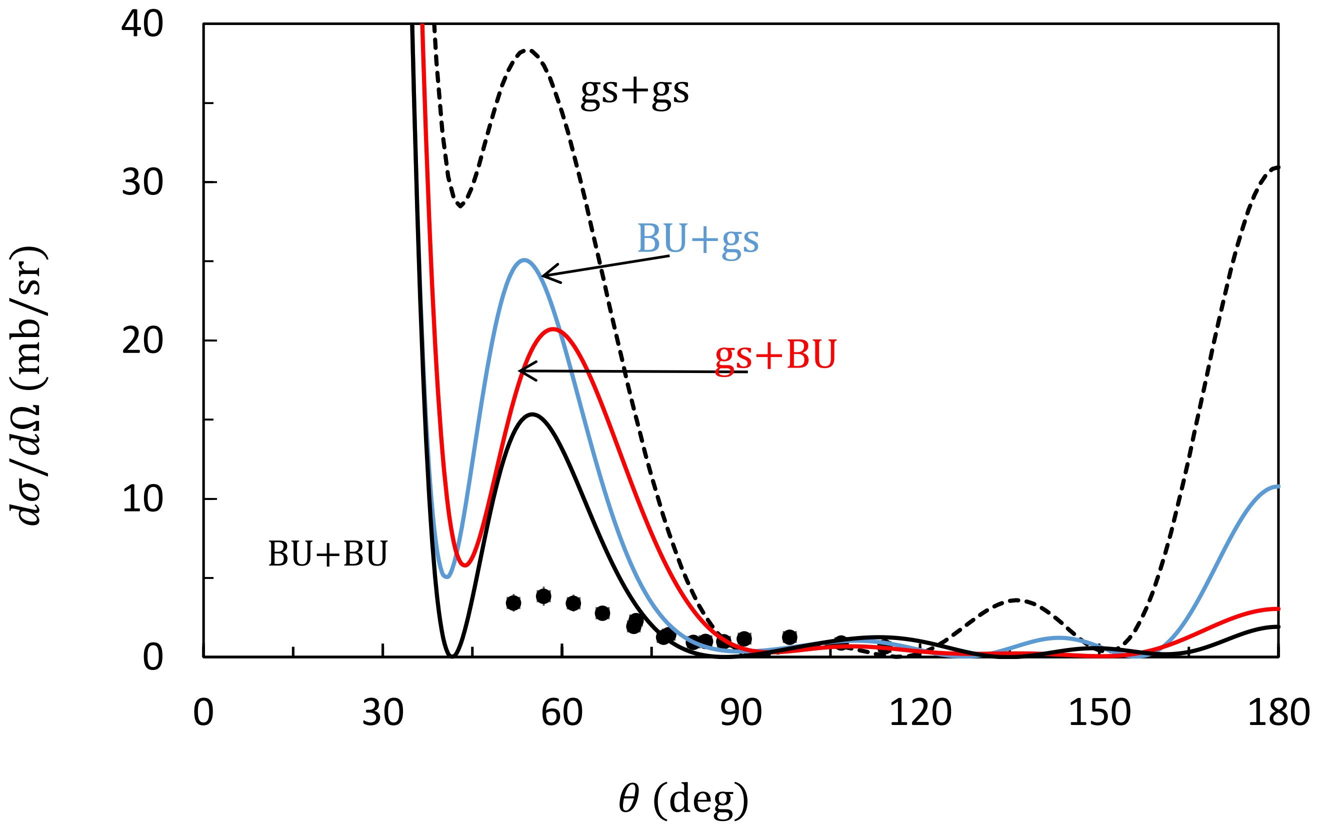
Convergence with respect to deuteron BU
(^{11}Li in ground state)



6. Results on ^{11}Li+d

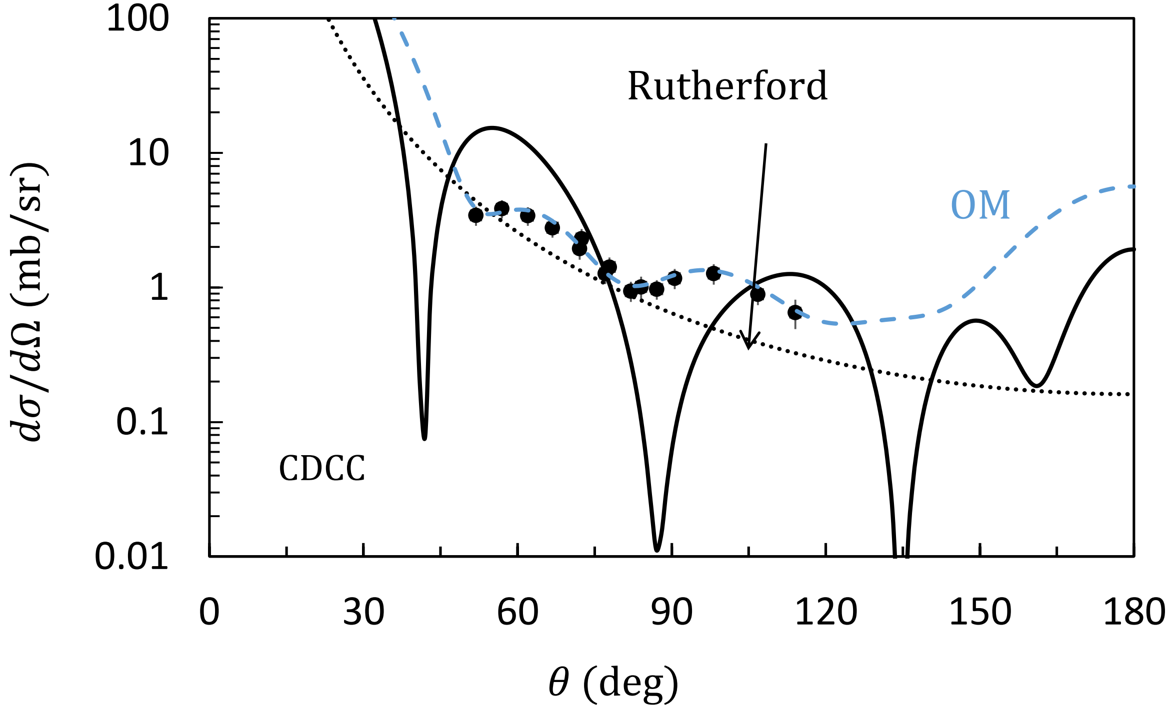
Second calculation: 5-body CDCC calculation

Summary

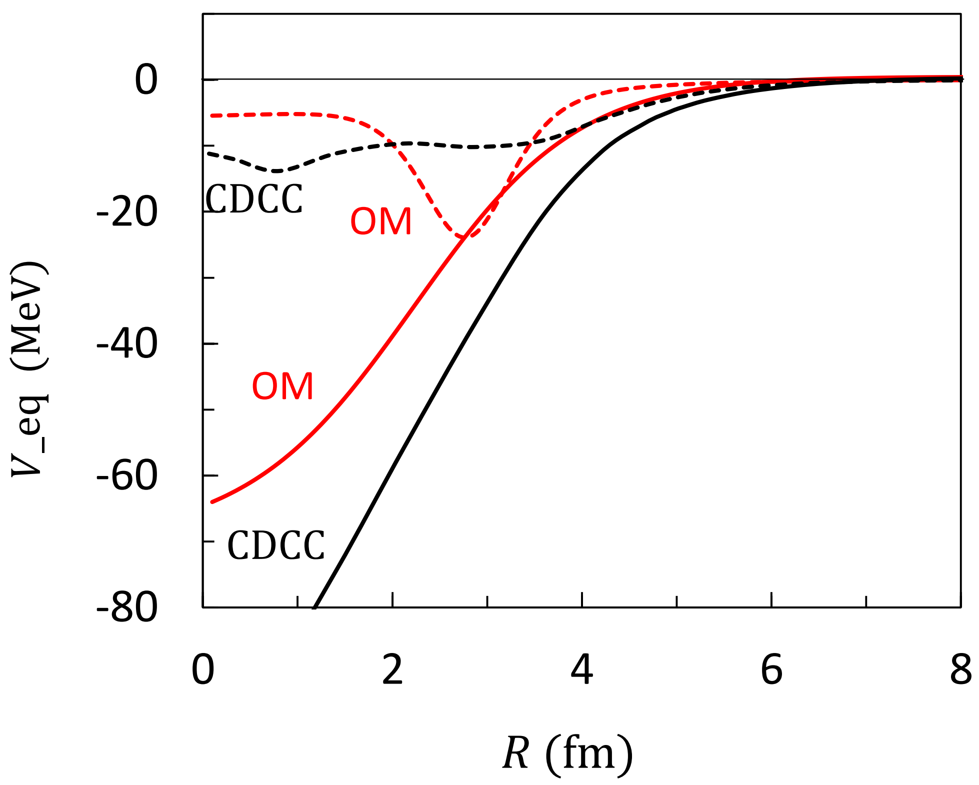


6. Results on ^{11}Li+d

Second calculation: 5-body CDCC calculation



Equivalent potential



- OM: optical potential fitted by Kanungo et al.
- Data are close to Rutherford scattering

A short range is necessary (surprising…)

Conclusions

7. Conclusion

^{11}Li structure

- Presence of a 1^- resonance at low energies
- No isoscalar character (as suggested by Kanungo et al.)

^{11}Li+p: 4-body CDCC

- Break up effects important for $\theta > 90°$ (also closed channels)
- Fair description of the elastic scattering cross section

^{11}Li+d: 5-bocy CDCC

- Extension of CDCC to “3+2” systems: important computer times (coupling potentials, large CC systems)
- “Full” convergence cannot be achieved
- Data surprisingly close to Rutherford → to be confirmed

Limitations of CDCC: ^{9}Li+p/n at E~5 MeV

- presence of resonances?
- treatment of Pauli forbidden states?

Data needed

- ^{9}Li+p elastic scattering → optical potential
- ^{11}Li+p/d data at higher energies