Low-energy $^{11}\text{Li}$ + p and $^{11}\text{Li}$ + d scattering in a multicluster model

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3. Overview of CDCC (Continuum Discretized Coupled Channels)
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Ref.: P. D., Phys. Rev. C, accepted
1. Introduction

Two recent experimental papers on $^{11}\text{Li} + p$ and $^{11}\text{Li} + d$ (elastic and inelastic scattering)

  
  $E_{\text{lab}} = 66$ MeV, $E_{\text{cm}} = 5.5$ MeV
  
  $\rightarrow$ dipole resonance in $^{11}\text{Li}$ at $E_x = 0.80$ MeV ($\Gamma = 1.1$ MeV)

- $^{11}\text{Li} + d$: R. Kanungo et al., PRL 114, 192502 (2015)
  
  $E_{\text{lab}} = 55.3$ MeV, $E_{\text{cm}} = 8.51$ MeV
  
  $\rightarrow$ dipole resonance in $^{11}\text{Li}$ at $E_x = 1.03$ MeV ($\Gamma = 0.5$ MeV), isoscalar character

E1 operator

Long wavelength approximation: $M_{\mu}^{E1} = \sum_i \left( \frac{1}{2} - t_{iz} \right) (r_i - R_{cm})_\mu$

Beyond the long wavelength approximation:

$$M_{\mu}^{E1} = \sum_i \left( \frac{1}{2} - t_{iz} \right) r'_{i\mu} \left( 1 - \frac{1}{10} (k\gamma r'_i)^2 + \cdots \right) + \cdots$$

$\Rightarrow$ isoscalar transitions are possible
1. Introduction

Present work


$^{11}\text{Li}+p$, $^{11}\text{Li}+d$ with the CDCC method

$^{11}\text{Li}+p$: 4-body CDCC (3+1)  
(see also T. Matsumoto et al., PTEP 2019, 126)

$^{11}\text{Li}+d$: 5-body CDCC (3+2)
2. Structure of $^{11}\text{Li}$
2. Structure of $^{11}\text{Li}$

- $V_{nn}$=Minnesota potential
- $V_{9\text{Li}+n}$=Woods-Saxon fitted on the scattering length
- Spin of the $^9\text{Li}$ core is neglected
- Forbidden states for $s_{1/2}$ and $p_{3/2}$→ removed by a supersymmetric transformation

$J=0^+$
- Bound state at $E_B=-0.378$ MeV
- $\sqrt{\langle r^2 \rangle}=3.12$ fm, exp=3.16±0.11 fm

$J=1^-$: $^9\text{Li}+n+n$ phase shifts (3-body phase shifts)

$\Rightarrow$ Dipole resonance near $E_{cm}=0.6$ MeV, $E_x=1.0$ MeV
2. Structure of $^{11}$Li

E1 transitions

$$B(E1, J_i \rightarrow J_f) = \frac{2J_f + 1}{2J_i + 1} |<\psi_{J_f}||M^{E1}\|\psi_{J_i}>|^2$$

2 options for $M^{E1}_\mu$: LWA $\rightarrow$ isoscalar=0

beyond the LWA $\rightarrow$ isoscalar $\neq 0$

- Peak near $E_{cm}$=0.6 MeV
  Consistent with the phase shifts

- Weak influence of high-order terms in $M^{E1}_\mu$
  Term $\sim \frac{1}{10} (k_\gamma r)^2$ with $k_\gamma = (E_{cm} + 0.4)/\hbar c$
  Even if $r^2$ is large the correction is quite small

- No isoscalar character for the transition
Overview of CDCC: Continuum Discretized Coupled Channel method
3. Overview of CDCC

• Introduced in the 70’s to deal with deuteron scattering
  Low binding energy of the deuteron → breakup is important

• Two-body projectile, three-body problem
  \[ H = H_0(r) - \frac{\hbar^2}{2\mu} \Delta_R + V_{t1}(R, r) + V_{t2}(R, r) \]

• \( H_0(r) \) = Hamiltonian associated with the projectile

• \( V_{t1}, V_{t2} \) = optical potentials between the target and the fragments
  (high energies: above the resonance region)
3. Overview of CDCC

- Projectile breakup described by approximate (discrete) states: \( H_0 \Phi_n^l m(r) = E_n^l \Phi_n^l (r) \)

Example: d=p+n

![Graph showing energy levels](image)

- **Pseudo state (PS):** \( E_n^l > 0 \)
  - Simulate breakup effects
  - No physical meaning
  - Depend on the basis

- **Ground state:** \( E_n^l < 0 \)
  - Physical
  - Does not depend on the basis

- CDCC well adapted to exotic nuclei (low binding energy)
  Example: \(^{11}\text{Be} = ^{10}\text{Be} + n\) (0.5 MeV)

- Low BU energy is not necessary! But BU effects are expected to be more important
3. Overview of CDCC

Extensions: same principle: discretization of the continuum

- 3-body projectiles: $^6\text{He}$, $^{11}\text{Li}$, $^{9}\text{Be}$
  - T. Matsumoto et al., PRC70 (2004) 061601
  - M. Rodriguez-Gallardo et al., PRC77 (2008) 064609

- A-body projectiles: $^7\text{Li}$, $^6\text{He}$, $^8\text{Li}$
  - Based on nucleon-target potentials \(\rightarrow\) no parameter
  - Y. Sakuragi et al., PTP Supp. 89 (1986) 136
  - P.D., M. Hussein, PRL 111 (2013) 082701

- 2-body projectile + 2-body target: $^{11}\text{Be}+\text{d}$, $^7\text{Li}+\text{d}$
  - Pseudostates in the projectile and in the target \(\rightarrow\) many channels
3. Overview of CDCC

- CDCC equations for $^{11}\text{Li}+p$ and $^{11}\text{Li}+d$

$^{11}\text{Li}+p$: 4-body

$^{11}\text{Li}+d$: 5-body

Total Hamiltonian: $H = H_1(x, y) + H_2(r) + T_R + \sum_{ij} U_{ij}(R, x, y, r)$

With $H_i =$ internal Hamiltonian of nucleus $i$  
$T_R =$ relative kinetic energy  
$U_{ij}(s) =$ optical potential between fragments $i$ and $j$

Then: standard CDCC procedure
3. Overview of CDCC

Standard CDCC procedure:

1. **Step 1:** solve $H_1 \Phi_{1k}^{jm} = E_1^{jk} \Phi_{1k}^{jm}$ for $^{11}$Li (hyperspherical coordinates)
   
   $H_2 \Phi_{2k}^{jm} = E_2^{jk} \Phi_{2k}^{jm}$ for $d$

   With $\Phi_{1k}^{jm}$ expanded on a basis (Lagrange functions: matrix elements are simple)
   
   → negative energies = **physical states**
   
   positive energies = **pseudostates**=(discrete) approximations of the continuum in 1 and 2

2. **Step 2:**

   Define channel functions: $\varphi_c(x, y, r, \Omega_R) = \left[\Phi_{1k_1}^{j_1} (x, y) \otimes \Phi_{2k_2}^{j_2} (r)\right]^I \otimes Y_L(\Omega_R)$

   with

   $I =$ channel spin

   $L =$ angular momentum between $d$ and $^{11}$Li

   index $c = (j_1, k_1, j_2, k_2, I, L)$

   and expand the total wave function as $\Psi^{JM\pi} = \sum_c u_c^{JM\pi}(R) \varphi_c(x, y, r, \Omega_R)$

   $u_c^{JM\pi}(R)$ to be determined
3. Step 3

Compute matrix elements of the potential \( \sum_{ij} U_{ij}(R, x, y, r) \)

\[
V_{cc'}^J(R) = \langle \varphi_c \mid \sum_{ij} U_{ij}(R, x, y, r) \mid \varphi_{c'} \rangle
\]

= integrals over 11 coordinates (8 angles + 3 radii): 5 analytical + 6 numerical integrals
(use of the Raynal-Revai coefficients)
4. **Step 4**: Solve the coupled-channel system

\[
\begin{bmatrix}
-\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dR^2} - \frac{L(L + 1)}{R^2} \right) + E_c - E
\end{bmatrix} u_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R) u_{c'}^{J\pi}(R) = 0
\]

- Standard coupled-channel system (general form common to most scattering theories)
- At large distances (only Coulomb): \( u_c^{J\pi}(R) \to I_c(R) \delta_{c\omega} - O_c(R) U_{c\omega}^{J\pi} \) (\( \omega \) = entrance channel)
  
  \( U_{c\omega}^{J\pi} \) = scattering matrix: provides the cross sections (elastic, inelastic, breakup, etc.)
- Solved with the **R-matrix method** (space divided in an internal and an external regions)
- The system must be solved for each \( J\pi \)
- Problems:
  - Many channels \( c \) (up to 9000 for \(^{11}\)Li+d)
  - Many \( J\pi \) values (depends on energy)
  - Long range of the potentials \( V_{cc'}^{J\pi}(R) \) (due to Coulomb)
  - Long calculations + many tests

5. **Step 5**

Determining the cross sections from the scattering matrices

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3. Overview of CDCC
The R-matrix method
4. The R-matrix method

Scattering matrix determined from the R-matrix theory

**R-matrix theory**: based on 2 regions (channel radius $a$)

- Lane and Thomas, Rev. Mod. Phys. 30 (1958) 257
- P.D., Computer Physics Communications 200 (2016) 199

Internal region: $R \leq a$

- Full Hamiltonian:
  
  $$ u^{I\pi}_c(R) = \sum_{i=1}^{N} c_i \phi_i(R) $$

  $\phi_i(R)$=Lagrange functions typically $N \sim 50$

External region: $R \geq a$

- Only Coulomb (monopole)
  
  $$ V^{I\pi}_{cc'}(R) = \frac{Z_p Z_t e^2}{R} \delta_{cc'} $$

  $$ u^{I\pi}_c(R) $$ has its asymptotic form

  $$ u^{I\pi}_{c,ext}(R) = I_c(kR) \delta_{c\omega} - \mathcal{O}_c(kR) U^{I\pi}_{c\omega} $$

- Main ingredient: matrix elements of the coupling potentials $V^{I\pi}_{cc'}(R)$: $< \phi_i | V^{I\pi}_{cc'} | \phi_j > \rightarrow$ fast method needed
- Matching at $R=a$ provides: scattering matrices $U^{I\pi} \rightarrow$ cross sections
4. The R-matrix method


- Gauss approximation: $\int_0^a g(x) dx \approx \sum_{k=1}^N \lambda_k g(x_k)$
  - N = order of the Gauss approximation
  - $x_k$ = roots of an orthogonal polynomial $P_N(x)$, $\lambda_k$ = weights
  - If interval $[0,a]$: Legendre polynomials
    $[0,\infty]$: Laguerre polynomials

- Lagrange functions for $[0,1]$: $f_i(x) \sim \frac{P_N(2x-1)}{(x-x_i)}$
  - $x_i$ are roots of $P_N(2x-1) = 0$
  - with the Lagrange property: $f_i(x_j) = \lambda_i^{-1/2} \delta_{ij}$

- Matrix elements with Lagrange functions: Gauss approximation is used
  \[
  < f_i | f_j > = \int f_i(x)f_j(x)dx \approx \delta_{ij}
  \]
  \[
  < f_i | T | f_j > \text{ analytical}
  \]
  \[
  < f_i | V | f_j > = \int f_i(x)V(x)f_j(x)dx \approx V(x_i)\delta_{ij} \Rightarrow \text{ no integral needed}
  \]

Also applicable to non-local potentials
Results on $^{11}\text{Li}+p$
5. Results on $^{11}\text{Li}+\text{p}$

a. Conditions of the calculations

Interactions
- $\text{n+p: Minnesota}$
- $^{9}\text{Li}+\text{p: Koning-Delaroche, Chapel Hill}$

Channel radius
$a\sim 25$ fm (stability tests)

$^{11}\text{Li}$ pseudostates
$E_{\text{max}} = 10$ MeV, $j_{\text{max}} = 3$
5. Results on $^{11}\text{Li}+p$

b. Convergence of the elastic cross section, $E_{\text{lab}}=66$ MeV, $E_{\text{cm}}=5.5$ MeV

$^{11}\text{Li}+p$

- $d\sigma/d\sigma_R$
  - $\theta$ (deg)

PS up to $E_{\text{max}}=5.5$ MeV → Closed channels are neglected
5. Results on $^{11}$Li+p

c. Comparison with experiment

OM: optical model with global parametrizations (KD03, CH89)

Smaller basis for $^{11}$Li Will be used for $^{11}$Li+d
5. Results on $^{11}$Li+p

d. Equivalent potentials

Question: can we find a single-channel equivalent potential?

• **J-dependent potential**

For the elastic channel: $(T_R + V_{11}^J(R) - E)u_1^J(R) = -\sum_{c \neq 1} V_{1c}^J(R)u_c^J(R)$

Equivalent to $(T_R + V_{11}^J(R) + V_{pol}^J(R) - E)u_1^J(R) = 0$

with $V_{pol}^J(R) = -\frac{\sum_{c \neq 1} V_{1c}^J(R)u_c^J(R)}{u_1^J(R)}$

Problems: J dependent
contains singularities (nodes of the wave function)

→ Construction of a J-independent potential
5. Results on $^{11}\text{Li}+p$

b) J-independent potential


$$V_{\text{pol}}(R) = \frac{\sum J V^J_{\text{pol}}(R) \omega^J(R)}{\sum J \omega^J(R)}$$

With $\omega^J(R)$=weight function

$$\omega^J(R) = (2J + 1) \left(1 - |U^J_{11}|^2\right) |u^J_1(R)|^2$$

reduces the influence of the nodes
gives more weight to the dominant J-values
Test: verify that $V_{\text{pol}}(R)$ reproduces the full calculation
5. Results on $^{11}\text{Li}+p$

![Graph showing potentials and differential cross sections for $^{11}\text{Li}+p$ interaction.](image)

- **Black**: CDCC equivalent potential
- **Red**: KD $^{11}\text{Li}+p$ potential

→ Range larger for CDCC
5. Results on $^{11}\text{Li}+\text{p}$

$^{11}\text{Li}+\text{n}$ potential (used for $^{11}\text{Li}+\text{d}$)

![Graph showing $V_{\text{eq}}(\text{MeV})$ vs $R$ (fm) and $d\sigma/d\Omega$ (mb/sr) vs $\theta$ (deg).]
Results on $^{11}\text{Li}+d$
6. Results on $^{11}\text{Li}+d$

- Main goal: simultaneous study of $^{11}\text{Li}+p$ and $^{11}\text{Li}+d$ (same conditions)
- Much more difficult: many channels, coupling potentials require long computer times, etc
  $\rightarrow$ no full convergence

First calculation: « standard » CDCC calculation with $^{11}\text{Li}+p/n$ equivalent potentials (deuteron BU only)
6. Results on $^{11}\text{Li}+d$

**Second calculation:** 5-body CDCC calculation

Convergence with respect to $^{11}\text{Li} \text{ BU}$
(d in ground state)

Convergence with respect to deuteron BU
($^{11}\text{Li}$ in ground state)
6. Results on $^{11}\text{Li}+d$

Second calculation: 5-body CDCC calculation

Summary
6. Results on $^{11}$Li+d

Second calculation: 5-body CDCC calculation

- OM: optical potential fitted by Kanungo et al.
- Data are close to Rutherford scattering

A short range is necessary (surprising...)

Equivalent potential

$d\sigma/d\Omega$ (mb/sr) vs $\theta$ (deg)

$\sigma/\Omega$ (mb/sr) vs $\theta$ (deg)

$V_{eq}$ (MeV) vs $R$ (fm)
7. Conclusion

\(^{11}\text{Li structure}\)
- Presence of a 1\(^{-}\) resonance at low energies
- No isoscalar character (as suggested by Kanungo et al.)

\(^{11}\text{Li+p: 4-body CDCC}\)
- Break up effects important for \(\theta > 90^\circ\) (also closed channels)
- Fair description of the elastic scattering cross section

\(^{11}\text{Li+d: 5-bocy CDCC}\)
- Extension of CDCC to “3+2” systems: important computer times (coupling potentials, large CC systems)
- “Full” convergence cannot be achieved
- Data surprisingly close to Rutherford \(\rightarrow\) to be confirmed

Limitations of CDCC: \(^9\text{Li+p/n at E} \sim 5\text{ MeV}\)
- presence of resonances?
- treatment of Pauli forbidden states?

Data needed
- \(^9\text{Li+p elastic scattering} \rightarrow\) optical potential
- \(^{11}\text{Li+p/d data at higher energies}\)