Inclusive breakup of $^{209}$Bi($^{6}$Li, $\alpha X$) and related topics

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Why $^6\text{Li}$

**Experiment**
- $^6\text{Li}$ is stable: high beam intensity
- High accuracy data exist for elastic scattering, breakup, fusion, and incomplete fusion

**Theory**
- $^6\text{Li}$ has two body cluster structure
- $^6\text{Li}$ induced reaction can be analyzed by a three body model
Problem to solve

- Theoretical prediction of alphas
- Not only integrated cross section, but also angular distribution


• Find the relation between breakup and fusion
• Explain the fusion suppression of weakly bound induced reaction
• Predict the fusion cross section
Inclusive breakup of $^6Li$ induced reaction

**Experimental point of view**
- $^6Li$ induced reaction
- Only alpha is detected

**Theoretical point of view**
- Contributions of alphas

\[
^6Li + ^{209}\text{Bi} \rightarrow \alpha + d + ^{209}\text{Bi}
\]

- $3b$ elastic breakup
- $4b$ elastic breakup
- $3b$ nonelastic breakup
- $4b$ nonelastic breakup
- Transfer
- Incomplete fusion
- Absorption
- Solve the problem by using a three body model (alpha+d+209Bi)
Experimental examples of inclusive breakup

\[ ^{6}\text{Li}(d+\alpha) \text{ induced reaction} \]

\[ ^{7}\text{Li}, ^{6}\text{Li}, t,d,p \]

\[ \Delta E \text{ (MeV)} \]

\[ E_{\text{total}} \text{ (MeV)} \]


\section{Surrogate reaction}

\begin{itemize}
  \item Andrew Ratkiewicz’s talk on April 7th
  \item Jutta Escher’s talk on June 11th
\end{itemize}

\begin{itemize}
  \item Desired Reaction
  \item Surrogate Reaction
  \item \( p \rightarrow \text{d, } n \rightarrow A \)
  \item \( B^{+} \)
\end{itemize}

\section{Knockout reaction}

\begin{itemize}
  \item Angela Bonaccorso’s talk on April 30th
  \item Study the Spectroscopic factor
  \item Current theory based on semi-classical (eikonal approximation)
  \item Fully quantum model is needed
\end{itemize}
Theoretical models for inclusive (nonelastic) breakup

• Requires inclusion of all possible processes through which the breakup fragment can interact with the target. Impractical in most cases.

In 1980s

• Ichimura, Austern, and Vincent developed a spectator-participant model (post-form)

• Udagawa and Tamura suggested a breakup-fusion model (prior-form)

• Hussein and McVoy adopted a spectator model with the Feshbach projection method

• Three different approaches with different predictions

Goals

• Find a suitable model for inclusive breakup
• Explore relations between these models

Challenges

• Numerically difficult
• No numerical implementation in 1980s-2000s even for Finite Range DWBA
The Ichimura, Austern, Vincent (IAV) model

- A three body model:
  - \( a + A \rightarrow b + \text{anything} \rightarrow (x+A)^* \)

Any possible states between \( x \) and \( A \) (including all nucleons degree of freedom)
Two body scattering with an optical potential

\[(x+A)^* \rightarrow x+A\]

**Key point**
relative wave function between \(x\) and \(A\)

**elastic scattering**

cross section: related to asymptotic part

\[\sigma_{el} = \frac{\pi}{k^2} \sum_{L=0}^{\infty} (2L + 1) \left| 1 - S_L \right|^2\]

**nonelastic scattering**
(absorption/reaction)

\[\sigma_A = \frac{2}{\hbar v} \frac{4\pi}{k^2} \sum_L (2L + 1) \int_0^\infty [-W(R)] \left| \chi_L(R) \right|^2 dR\]

**Key point**
relative wave function between \(x\) and \(A\)
The Ichimura, Austern, Vincent (IAV) model

• A three body model:
  • \( a + A \rightarrow b + \text{anything} \rightarrow (x+A)^* \)

Any possible states between \( x \) and \( A \) (including all nucleons degree of freedom)

• Relative wave function between \( x \) and \( A \) in three body reaction with optical potentials

\[
(E_x - K_x - U_x) \varphi_x(k_b, r_x) = \langle r_x \chi_b(k_b) \mid V_{\text{post}} \mid \Psi^3_b \rangle
\]

\[
\varphi_x^\ell(k_b, r_x) \xrightarrow{r \to \infty} - S_\ell \mathcal{H}^+_l
\]

• Elastic breakup: equivalent to CDCC (three body scattering)

\[
\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_x} = \frac{(2\pi\hbar)^3}{\mu_x^2 v_x} \rho_b(E_b) \rho_x(E_x) \left| f\left(\hat{k}_b, \hat{k}_x\right)\right|^2
\]

\[
f\left(\hat{k}_b, \hat{k}_x\right) = - \sum C \mathcal{Y}^{m_b}_l(\hat{k}_b) \mathcal{Y}^{m_x}_l(\hat{k}_x) i^{-l_x} S_\ell
\]

• Nonelastic breakup: (absorption)

\[
\frac{d^2\sigma}{dE_b d\Omega_b} = - \frac{2}{\hbar v_a} \rho_b(E_b) \langle \varphi_x(\hat{k}_b) \mid W_x \mid \varphi_x(\hat{k}_b) \rangle
\]
Effective two body interactions

- Effective two body interaction in three body model
- Testing the effective interactions in CDCC model

$\Psi^{3b(+)} \simeq \Psi^{CDCC(+)}(r_a, r_{bx}) = \sum_i \phi_i^a(r_{bx}) \chi^{i(+)}_a(r_a) + \sum_c \phi_c^a(k_c, r_{bx}) \chi^{c(+)}_a(K_c, r_a)$

- Compare the elastic scattering cross section with the data

Inclusive breakup of $^{209}$Bi($^6$Li,$\alpha$X)

- $^6$Li→($\alpha$ + d), $S(d)$=1.474 MeV

- Only $\alpha$ is detected

- EBU : CDCC (FRESCO)
- NEB : IAV model
  - DWBA $\Psi^{3b(+)} \simeq \Psi^{\text{DWBA}(+)} = \chi^{(+)}_a \phi_a$

- Total Breakup (TBU)=EBU+NEB

- Dominated by NEB
- EBU has large contributions at small angles
- Supports IAV model
  
Breakup and fusion

- From the barrier penetration picture
  - [Diagram: tightly bound nuclei vs. weakly bound nuclei]

- Complete Fusion is suppressed due to weak binding of the projectile

- Complete fusion: total charge of the projectile is absorbed by the target
- Incomplete fusion: part of the projectile is absorbed by the target

- Challenges
  - To correctly understand fusion suppression (not only from semi-classical picture) and simultaneously predict the complete fusion cross section
  - To study incomplete fusion is breakup-fusion (two-step) or transfer to continuum (one-step)
Study the fusion cross section through a three body model

- Take $^6\text{Li}+A$ as an example

\[ \sigma_R \approx \sigma_{\text{CF}} + \sigma_{\text{EBU}} + \sigma_{\text{NEB}}^{(b)} + \sigma_{\text{NEB}}^{(x)} \]
Study the fusion cross section through a three body model

\[ \sigma_{\text{CF}} \approx \sigma_R - \sigma_{\text{EBU}} - \sigma_{\text{NEB}}^{(b)} - \sigma_{\text{NEB}}^{(x)} \]

- Apply the above relation to \(^6\text{Li}+^{209}\text{Bi}\) reaction around the Coulomb barrier

- Compare calculated fusion cross section with experiment

- EBU mechanism plays a minor role

- Dominant breakup mechanism in both reactions is alpha production due to \((^6\text{Li},\alpha X)\) NEB.


Unraveling the mechanisms leading to fusion suppression

K. J. Cook

What is the mechanism for suppression of complete fusion?

• Weak binding leads to strong clustering → Displacement of clusters from the center of mass.

• This makes the triton amenable to transfer → ICF

• Requires the center of mass of the $^7\text{Li}$ projectile to get closer to the target so that the entire projectile fuses. → CF suppression

• Numerical support from Lei & Moro PRL 122 042503 (2019) using the Ichimura, Austern, and Vincent (IAV) spectator-participant inclusive breakup model. They associate it with a "Trojan Horse" mechanism.

• No-capture breakup has larger b

• Breakup-capture has smaller b

• Requires the center of mass of the projectile to get closer to the target so that the entire projectile fuses.

Taken from Kaitlin's talk

\[ \theta \]
\[ b \]
Unraveling the mechanisms leading to fusion suppression

When the binding energy becomes larger, the calculated cross section approaches the barrier penetration model (BPM).


Use a toy model to study effects of separation energy
- vary the binding energy of $^6\text{Li}(\alpha+d)$ in the projectile.

When the binding energy becomes larger, the calculated cross section approaches the barrier penetration model (BPM)

Why NEB is so important

The NEB of α production

\[ ^{6}\text{Li} + ^{209}\text{Bi} \rightarrow \alpha + (d + ^{209}\text{Bi})^* \]

Compare with

\[ d + ^{209}\text{Bi} \rightarrow (d + ^{209}\text{Bi})^* \]

Trojan Horse Effect

Incomplete fusion is part of the projectile absorbed by the target.

By definition, ICF is part of NEB, for example:

$$
\sigma(6\text{Li}+209\text{Bi}\rightarrow \alpha + 211\text{Po}^*) = \sigma(\text{NEB}) - \\
\sigma(6\text{Li}+209\text{Bi}\rightarrow d+\alpha + 209\text{Bi}^*) - \\
\sigma(6\text{Li}+209\text{Bi}\rightarrow n+p+\alpha + 209\text{Bi}) - \\
\sigma(6\text{Li}+209\text{Bi}\rightarrow n+p+\alpha + 209\text{Bi}^*) - \text{others}
$$

that we describe the massive transfer reaction as a two-step process. Take again the above example. The first step is then the breakup of $^{14}\text{N}$ into $\alpha + ^{10}\text{B}$. This is then followed by the second step, in which $^{10}\text{B}$ is fused into $^{158}\text{Tb}$.

suggested two step process

suggested one step process
Exploring the reaction path for incomplete fusion

Incomplete fusion: part of the projectile absorbed by the target

Two-step: projectile is inelastically excited into its continuum and then fuses with the target

One-step: fragment fuses with the target directly from its ground state

Resolve this puzzle by studying nonelastic breakup (incomplete fusion is a part)

Use CDCC wave-function in the IAV model:

\[
\psi_{CDCC}(r_a, r_{bx}) = \sum_b \phi^b(r_{bx}) \chi^{b(+)}(r_a) + \int d\mathbf{k} \phi^k(r_{bx}) \chi^{k(+)}(r_a)
\]

- Continuum and ground states are separated
- Allows to study continuum effects on the NEB
- Test validity of DWBA
Apply to $^6\text{Li}$ induced reaction

\[ \psi_{\text{CDCC}(\text{full})}\left(\mathbf{r}_a, \mathbf{r}_{bx}\right) = \sum_b \phi^{b}\left(\mathbf{r}_{bx}\right) \chi^{b(+)}(\mathbf{r}_a) + \int d\mathbf{k}\phi^{k}\left(\mathbf{r}_{bx}\right) \chi^{k(+)}(\mathbf{r}_a) \]

\[ \psi_{\text{CDCC}(\text{g.s.})}\left(\mathbf{r}_a, \mathbf{r}_{bx}\right) = \sum_b \phi^{b}\left(\mathbf{r}_{bx}\right) \chi^{b(+)}(\mathbf{r}_a) \]

\[ \psi_{\text{DWBA}(\text{full})}\left(\mathbf{r}_a, \mathbf{r}_{bx}\right) = \phi\left(\mathbf{r}_{bx}\right) \chi^{(+)}(\mathbf{r}_a) \]

- DWBA is a good approximation compared to CDCC
- Nonelastic breakup (incomplete fusion) is mixture of one-step (>90%) and two-step (<10%) processes

Summary and outlook

• Summary

  • Studied $^6$Li induced reactions
  
  • Found the reaction mechanism for complete fusion suppression
  
  • Investigated the reaction path for incomplete fusion (nonelastic breakup)

• Outlook

  • Find a suitable theory to extract incomplete fusion cross sections for deuteron (surrogate reaction) and $^6$Li
  
  • Apply the IAV model for knockout reaction to verify semi-classical model
  
  • Study uncertainties caused by effective interactions